Stability Issues in Recovering Illumination Distribution from Brightness in Shadows

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Abstract

This paper describes a robust method for estimating, in a reliable manner, the illumination distribution of a real scene from shadows in a given image. In general, shadows in a scene are caused by the occlusion of incoming light; image brightness inside shadows have a great potential for providing distinct clues to the illumination distribution of the scene. Taking advantage of this fact, we recently proposed to estimate the illumination distribution of a real scene from a single image of the scene. The proposed method has been applied successfully to real images with complex illumination distributions. Nevertheless, it was found that, under certain circumstances, the method sometimes failed to provide a correct estimate of illumination distribution. Those failures stem from the fact that the method does not take into account several factors regarding the stability of illumination estimation. The purpose of this study is to analyze how much information is obtainable from a given image about the illumination distribution of the scene. In particular, we carefully examine the source of instability of using shadows obtained from a single image for the estimation in several aspects: blocked view of shadows by the object; limited sampling resolution for image brightness inside shadows; and the appropriate light model to approximate the illumination distribution of the scene. Based on this analysis, we propose a new method that guarantees to estimate the illumination distribution of a scene in a reliable manner regardless of types of input images such as the shape of an occluding object or a camera position.

1 Introduction

The image brightness of a three-dimensional object is the function of the following three components: the distribution of light sources, the shape, and reflectance of a real object surface [5, 6]. Three major research areas in physics-based vision are derived from the relationship among them : shape-from-brightness (with a known reflectance and illumination); reflectance-from-brightness (with a known shape and illumination); illumination from brightness (with a known shape and reflectance).

The first two kinds of analyses, *shape-from-brightness* and *reflectance-from-brightness*, have been intensively studied using the shape from shading method [4, 7, 8, 18], as well as through reflectance analysis research [1, 9, 11, 13, 16, 22].

In contrast, relatively limited amounts of research have been conducted in the third area, illumination-from-brightness [3, 7, 12, 15, 17, 19, 25]. This is because real scenes usually include both direct and indirect illumination distributed in a complex way and it is difficult to analyze characteristics of the illumination distribution of the scene from image brightness. Most of the previously proposed approaches were conducted under very specific illumination conditions, e.g. there is only one direct light source in the scene, and difficult to be extended for more natural illumination conditions, or multiple input images taken from different viewing angles were necessary.

Recently, we turned our attention to the information which shadows cast by an object in a scene could provide about the illumination distribution of the scene,[1] and introduced a method for estimating the illumination distribution of a real scene from shadows observed in a single image [20]. This method was later extended for more efficient estimation of illumination distribution by incorporating an iterative refinement scheme [21].

Our method has shown promising results for estimating illumination distribution in a real scene. Nevertheless, the question of how to examine whether a given input image provides sufficient information to robustly estimate the illumination distribution of the scene still remains to be addressed. In fact, it was found that, under certain circumstances, our method sometimes failed to provide a correct estimate of illumination distribution.

The instability of this method stems from the lack of consideration of the fact that shadow pixels observed in a single image provide only a limited amount of the information about the illumination distribution of a scene. Namely, an object in a scene occludes part of the view from a camera taking input images, and only part of the shadows cast by the object can be observed in the image. Also, the field of view of a camera taking an input image directly controls what size portions of shadows cast by an object can be observed in the input image. Furthermore, since the number of image pixels is limited, radiance distribution inside shadows can be measured only up to a certain resolution. Without taking those

[1] In the past, shadows have been mainly used for determining the 3D shapes and orientations of an object which cast shadows onto the scene [2, 10, 14, 23].

factors into consideration, estimation of illumination distribution of a real scene from a given image may not be performed reliably.

The purpose of this study is to present a method to analyze how much information is obtainable from a given input image for a particular direction of the entire illumination distribution. In particular, we carefully examine the source of instability of using shadows obtained from a single image for estimating the illumination distribution of a scene in several aspects: blocked view of shadows by the object; limited sampling resolution for image brightness inside shadows; and the appropriate light model to approximate the illumination distribution of the scene. Then, based on this analysis, a set of light sources that approximates the illumination distribution of the scene is defined from the given input image so that radiance values of those light sources can be estimated reliably regardless of types of input images such as the shape of an occluding object or a camera position. This new proposed approach based on the analysis in observed image brightness enlarges the variety of images to which the method can be applied.

The rest of the paper is organized as follows. In Section 2, we briefly describe the basic framework of our previously proposed method for estimating illumination distribution of a real scene from a single image. Then, in Section 3, we explain several problems regarding the stability of the estimation method. In Section 4, we provide solutions to overcome those problems, and report experimental results to demonstrate the effectiveness of the proposed solutions in Section 5. Finally, in Section 6, we present concluding remarks.

2 Overview of illumination estimation from shadows

Before we discuss each of the factors causing instability of illumination estimation, let us briefly present the overview of our previously proposed method for estimating illumination distribution of a scene from a given image of the scene [20].

In the following sections, we refer to the image with shadows as the *shadow image*, to the object which casts shadows onto the scene as the *occluding object*, and to the surface onto which the *occluding object* casts shadows as the *shadow surface*.

The illumination distribution of a scene is first approximated by discrete sampling of an extended light source in our method; whole distribution is represented as a set of point sources equally distributed in the scene as shown in Figure 1. Then total irradiance E at the $shadowsurface$ received from the entire illumination distribution is computed as

$$E = \sum_{i=1}^{n} L_i S_i \cos\theta_i \qquad (1)$$

where L_i $(i = 1, 2, ..., n)$ is the illumination radiance per

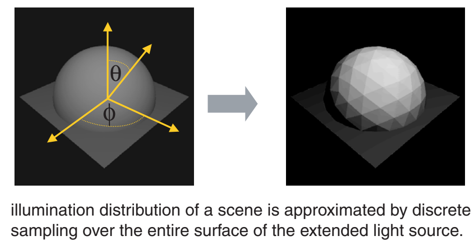

Figure 1: Approximation of the illumination distribution of a scene

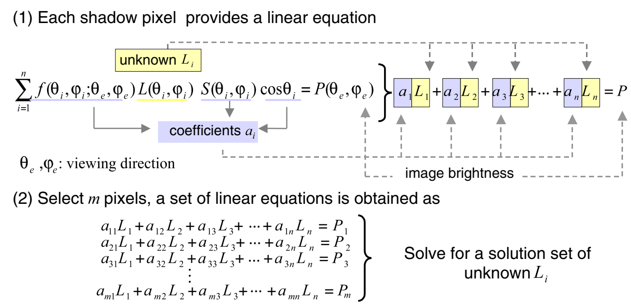

Figure 2: Outline of the previously proposed method

solid angle $\delta\omega = 2\pi/n$ coming from the direction (θ_i, ϕ_i), and S_i are occlusion coefficients. $S_i = 0$ if L_i is occluded by the *occluding object*, and $S_i = 1$ otherwise. Then, this approximation leads each image pixel inside shadows to provide a linear equation with unknown radiance of those sources as

$$a_1 L_1 + a_2 L_2 + a_3 L_3 + \cdots + a_{1n} L_n = P \qquad (2)$$

where P is the image brightness of the pixel, and the coefficients $a_i (i = 1, 2, ..., n)$ represent how much incoming L_i is reflected on the *shadow surface* towards the viewing direction. Here, a_i includes occlusion coefficients S_i, the bidirectional reflectance distribution function (BRDF) and cosine factors as illustrated in Figure 2 (1). These coefficients are computed from the 3D geometry of a surface point, the *occluding object* and the illuminant direction, and the surface reflectance property of the *shadow surface* that is either given or estimated.[2]

Finally, a set of linear equations is derived from the brightness changes observed in the *shadow image* and solved for a solution set of unknown radiance of L_i's in Equation 3.

[2] For modeling the shape of an *occluding object* from a *shadow image*, a modeling tool called the 3D Builder from 3D Construction Company [26] was used to obtain the experimental results shown in Section 5.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ & \cdots & \cdots & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \vdots \\ L_n \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_m \end{bmatrix}$$

$$A \cdot L = P \tag{3}$$

3 Stability issues in illumination estimation

Shadows are, in general, caused by occlusion of incoming light by an *occluding object*; brightness changes inside shadows have a potential for providing important clues to the illumination distribution of the scene. However, brightness distributions inside shadows vary as a function of the lighting environment of the scene and the shape of an *occluding object*. Furthermore, the amount of the information contained in a *shadow image* about the illumination distribution of a scene changes from image to image, depending on how much of the *shadow surfaces* are blocked by the *occluding object* and how much are covered by the field of view of the camera taking the *shadow image*.

Therefore, it is essential to evaluate an *shadow image* first in terms of how much information about the illumination distribution of the scene the image can provide. The lack of this evolution step is the main reason why our previously proposed method for illumination estimation sometimes failed to recover the illumination distribution of a scene from observed shadows. In this section, we explain two main factors that control the stability of the illumination estimation from shadows: blocked view of shadows; limited sampling resolution for radiance distribution inside shadows. Then, later in Section 4, we will introduce solutions for overcoming these two problems.

3.1 Distribution solvable from a given shadow image

Let us first consider the ideal case where radiance values of light source L_i's can be estimated without suffering from any instability problems.

As shown in Figure 2 (2), a set of linear equations is derived from a given *shadow image*. In Equation 3, the coefficients matrix A of the system $A \cdot L = P$ is consists of n column vectors $\vec{a_i} = (a_{1i}, a_{2i}, \ldots, a_{mi})$ where n is the number of sampling directions, and m is the number of the image pixels. Here, if the coefficients matrix A consist of n linearly independent column vectors $\vec{a_i}$, we numerically obtain enough constraints for solving A for unknown radiance of L_i's.

As has been noted, the amount of the information that shadows in a *shadow image* provide is limited. Under some circumstances, we might not be able to derive n linear independent vectors $\vec{a_i}$ from a given *shadow image*. Note that the coefficients a_i in Equation 2 represent $f(\theta_i, \phi_i; \theta_e, \phi_e) S_i \cos\theta_i$, and the most powerful clue to the illumination distribution of a scene among the components of a_i is S_i (either 0 or 1) since the other components do not change so much from image pixel to pixel. As a consequence, it makes the estimation more stable if we observe the difference between radiance of two shadow regions for each light source: one illuminated and the other not illuminated by the light source. In other words, the more variations in S_i's in shadow regions we have, the more likely it is that we will obtain n linearly independent column vectors, thereby making it possible to robustly estimate radiance values of the light sources approximating illumination distribution of the scene.

In the following sections, we describe the main sources that prevent us from observing both shadow and non-shadow regions for a light source in a particular direction. This tends to makes the matrix A ill-conditioned and leads to the instability problem of the proposed method.

3.2 Blocked view of shadows

As has been noted, the visible portion of a *shadow surface* changes from image to image depending on how wide the field of view of a camera taking the *shadow image* is, and what size portions of the view of the camera are occluded by the *occluding object*. Only part of the shadows cast by the occluding object can be usually observed in the *shadow image*, with the exception of special cases such as a camera aimed directly towards a floor onto which shadows are cast.

Consider the case shown in Figure 3 as an example. In this case, the region behind the *occluding object* cannot be seen by the camera. More precisely, only the shadow caused by the light source 3 can be observed in the *shadow image*, and shadows cast by the light sources 1 and 2 are not seen.

Let $\vec{a_1}$ and $\vec{a_2}$ as column vectors and L_1 and L_2 as radiance values for the light source 1 and 2 respectively, then in the case shown in this example, $\vec{a_1}$ and $\vec{a_2}$ become close to singular. In other words, the occlusion coefficients S_i of $\vec{a_1}$ and those of $\vec{a_2}$ are the same for all shadow pixels, and only differences for distinguishing the light source 1 from the light source 2 are in $f(\theta_i, \phi_i; \theta_e, \phi_e) cos\theta_i$. As a result, the estimation becomes too sensitive to the image noise, and radiance values of the light source 1 and 2 can not be reliably estimated.

3.3 Limited sampling resolution for radiance distribution inside shadows

Second, the number of image pixels is fixed, and therefore radiance distribution inside shadow regions can be measured only up to a certain resolution.

In the basic formulation of the previously proposed method described in Section 2, it was simply assumed that we could obtain a large enough number of image pixels inside shadow regions to provide sufficient constraints for

[3] For our current implementation, we solve the problem using the linear least square algorithm with non-negativity constraints (using a standard MATLAB function) to obtain an optimal solution with no negative components.

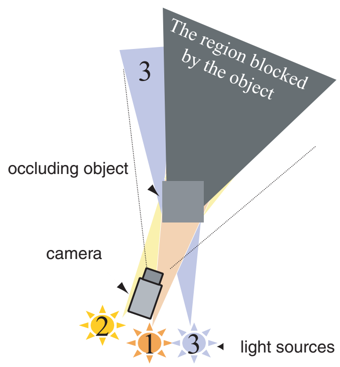

Figure 3: Blocked view of shadows

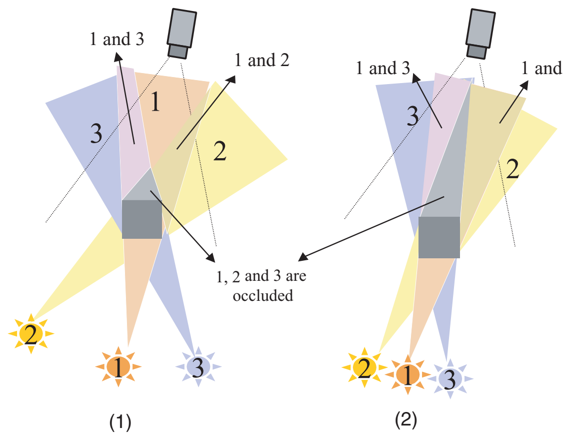

Figure 4: Difference in visible combinations of occlusions of lights: (1) coarsely distributed light sources and (2) densely distributed light sources

solving a set of linear equation (Equation 3) for unknown radiance values of point light sources. However, this is true only when we can measure radiance values inside shadows at any resolution.

To illustrate this point, consider two cases shown in Figure 4. In the first case in (1), illumination distribution of a scene is represented by coarsely distributed point light sources 1, 2, and 3. On the other hand, more densely distributed point light sources are used in the other case shown in (2). Shadows are partitioned into smaller regions depending on which light source is occluded by the *occluding object*. For instance, the shadow in the left case is partitioned into the following regions in order from left to right: 3, 1&3, 1, 1&2&3, 1&2, and 2.

As we can see in the case in Figure 4 (2), the shadow does not contain a region where only the point light source 1 is occluded. Also, partitioned regions such as 3 or 2 are significantly smaller than those in the case in Figure4 (1). As a result, we obtain less of a variety of partitioned regions inside shadows, thus making the estimation less reliable, and the estimation further becomes more sensitive to image noise because each partitioned region contains fewer image pixels.

Since the number of image pixels is limited, radiance distribution inside shadows can be measured only up to a certain resolution. As a result, sampling resolution of illumination distribution of a real scene is also bound to a certain limit. For instance, if we use more densely distributed point light sources for the estimation, we might not even observe any image pixels of some shadow regions.

It follows from the discussions above that there is a chance that the estimation becomes too unstable to provide a correct estimate of the illumination distribution of the scene for a given set of point light sources for the estimation. As a solution to this problem, a new procedure to evaluate an input *shadow image* in terms of the information about the illumination distribution of a scene obtainable from the image is considered in the next section. Using the outputs from this procedure, a set of point light sources is defined so that radiance values of those light sources can be estimated reliably from the given *shadow images*.

4 Techniques for robust illumination estimation

Recall the case shown in Figure 3 where two column vectors $\vec{a_1}$ and $\vec{a_2}$ of the light sources 1 and 2 respectively are close to singular. Here, if the light sources 1 and 2 are combined to form a larger light source for more coarse sampling of illumination distribution, we are able avoid facing the instability problem caused by the light source 1 and the light source 2. Similarly, the sampling resolution problem described in Section 3.3 can be avoided if we use the more coarse sampling instead of dense sampling of illumination radiance whenever it is decided to be necessary in terms of the stability of computation for solving the set of linear equations obtained from a given *shadow image*.

In this section, we will adopt a strategy for changing the sampling density of the illumination distribution depending on the amount of the information obtainable from a *shadow image* for a particular direction of the illumination distribution.

4.1 Selection of illumination distribution samplings

The derived coefficients matrix A from a *shadow image* is first examined in terms of the stability of computation for solving the set of linear equations obtained from a given *shadow image*.

Stability of a given system is measured by the condition number. Some systems are sensitive to errors and others are not. For a positive matrix A, the condition number c is computed (again using a standard MATLAB function) as

$$c = \sigma_{max}/\sigma_{min} \tag{4}$$

where σ_{max} is the maximum singular value of the matrix A, and σ_{min} is the minimum singular value.

If this condition number c is sufficiently small, Equation 3 can be solved without numerical instability. Therefore, the given set of light sources representing real illumination distribution is appropriate for the given *shadow image*. On the other hand, if the condition number c is large, the problem of illumination estimation for the given set of light sources is close to ill-conditioned. Therefore, we need to reduce light sources representing illumination distribution by combining several light sources into one with a larger solid angle.

Then, our question is how to select those light sources to be combined for sampling the illumination distribution more coarsely. This is done by examining the column vector $\vec{a_i} = (a_{1i}, a_{2i}, \ldots, a_{mi})$ corresponding to a light source L_i in Equation 3. If two vectors $\vec{a_i}$ and $\vec{a_j}$ for two light sources L_i and L_j are about the same, Equation 3 becomes ill-conditioned and brightness distribution inside shadows does not provide sufficient information for determining radiance of those two light sources. Therefore, those two light sources need to be combined to form a larger light source for more coarse sampling of illumination distribution. We evaluate the similarity of two vectors $\vec{a_i}$ and $\vec{a_j}$ with their dot product $\vec{a_i} \cdot \vec{a_j}$.

Since the entire stability computation depends on the coefficients matrix A and the radiance values L_i are estimated from these coefficients as well, it is important to compute a_i of the matrix A as accurately as possible. For this purpose, we further introduce the following two techniques. One is a technique for computing more accurate shadow coefficients S_i using the more appropriate light model for L_i. The other is a technique for sampling pixels from a *shadow image* in such a way as to maximize the information obtainable from the image.

4.2 Occlusion test of incoming lights

In the proposed approach, the entire illumination distribution of a scene is represented as a set of point sources equally distributed in the scene whose solid angle is defined by a sampling density n. Nevertheless, the occlusion tests are previously performed simply by examining occlusions of the center points of those light sources.[4]

The important point to note is that all objects casting shadows from extended light sources produce three types of regions: completely illuminated, completely occluded (*umbra*), and partially occluded (*penumbra*). As illustrated in Figure 5, a penumbra surrounds an umbra and there is always a gradual change in intensity from a penumbra and an umbra. The previous approach can treat only hard edged shadows correctly, and the approximation with point light sources introduces inaccuracy in illumination estimation in the presence of soft edged shadows, i.e., shadows with umbra and penumbra. Mixing up the umbra and the penumbra area not only prevents us from computing accurate coefficients a_i in Equation 2 but also reduces the varieties of occlusions observed in a *shadow image*.

[4] As described in Section 2, occlusion coefficients S_i in Equation 1 becomes 0 if the center point of L_i is occluded by the *occluding object*, and $S_i = 1$ otherwise.

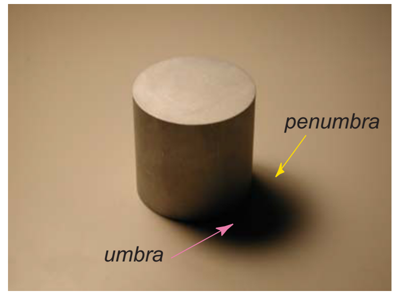

Figure 5: Umbra and penumbra

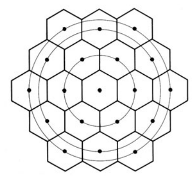

Figure 6: Hexagonal grids used for supersampling inside one light source

One approach to reduce the side-effect from this approximation is to increase the number of point light sources used for approximating illumination distribution and to compute occlusion coefficient S_i more accurately between 0 and 1, cf., 0 or 1.[5]

In our actual implementation of this super-sampling scheme, a hexagonally packed grid is defined inside each light source and used for computing the occlusion coefficients. The size of a hexagonally packed grid for a light source is determined from the solid angle of the light source [24]. Figure 6 shows a hexagonal grid, made up of 19 hexagons, used in our experiments. For each point inside the hexagonal grid, we determine whether a light ray coming from the point is occluded by an *occluding object*. Then, occlusion coefficient S_i for the light source is given as a ratio of unoccluded sampling points inside the hexagonal grid.

4.3 Pixel selection

For evaluating an input *shadow image* correctly, it is essential to select image pixels from a *shadow image* to maximize variation of patterns of occlusion of light sources by an *occluding object*.

First, the visible part of *shadow surface* is partitioned into clusters based on the combinations of occlusion coefficients

[5] In theory, any subtle change in illumination distribution can be approximated with sufficient accuracy if an infinite number of point light sources is used. However, the computational cost would increase prohibitively if too many light sources were to be used. Moreover, the use of too many point light sources causes the sampling resolution problem described in Section 3.3.

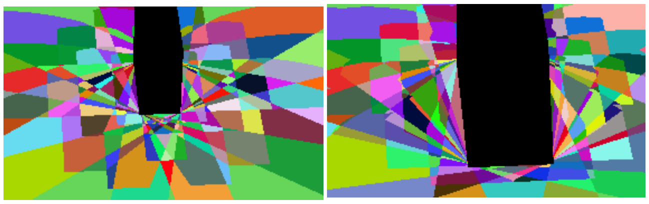

Figure 7: Clustering results

S_i. In other words, pixels that have the same combination of coefficients S_i are clustered into the same group. Conceptually, those clusters are similar to partitioned regions in shadows in Figure 4. At the same time, the number of pixels in one group is examined so that we can avoid selecting a pixel from a group that contains fewer image pixels.

Figure 7 shows several examples of partitioning based on the occlusion coefficients S_i with pseudo colors. Here each color represents an individual class with a different combination of occlusions of the light sources, and the block region corresponds to the *occluding objects*. From these examples of partitioning, we also see that patterns of partitioning differ from image to image depending on factors such as the shape of an *occluding object* and the camera viewpoint.

After the *shadow surface* is partitioned into clusters, one pixel is selected from each of those clusters. By selecting image pixels in this way, we can maximize variation of patterns of occlusion of light sources by an *occluding object*, and therefore, we are able to evaluate the input *shadow image* in an appropriate manner. In addition, we are able to avoid selecting redundant pixels, i.e., pixels that provide the same information about the illumination of the scene as other pixels.

5 Experimental results

We have tested our proposed method described in Section 4 by using real images taken in both a laboratory and an office. First, we have tested our proposed method by using images taken under a relatively simple illumination environment so that we can examine the performance of our proposed method carefully. Figure 8 shows our experimental setup with three light sources with different colors. The input images taken by using this setup with different *occluding objects* are shown in Figure 9 (1).

The results of illumination estimation with coarse samplings are shown in Figure 9 (2). Here, the estimated illumination radiance is visualized by mapping their values onto a dome shown in Figure 1. As we can see in these results, illumination distribution was correctly estimated for this set of light source samplings. However, after we increased the number of light sources for sampling illumination distribution, the estimation became unstable. Figure 9 (3) shows densely distributed light sources and in fact, we can see that the results of estimation shown in this column are erroneous

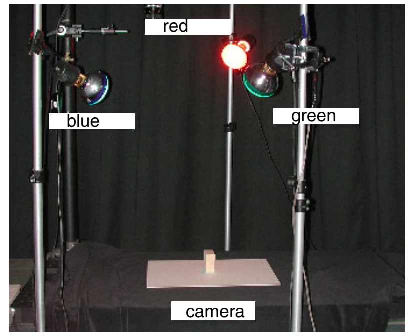

Figure 8: Experimental setup with three area light sources with different colors

especially around the blue and green light sources.

On the other hand, Figure 9 (4) shows the result of illumination estimation with our proposed method. Their stability measure is represented in the left of Figure 9 (4). Here, brighter light sources represent more reliable sampling regions which are not required to be merged to form a larger light source for more coarse sampling. Unlike the estimation results in Figure 9 (3), illumination distribution due to three light sources with different colors was correctly estimated in the right of Figure 9 (4).

Based on the estimated illumination distribution from the input *shadow image* shown Figure 10 (1), several synthetic objects were superimposed onto a synthetic surface in the bottom row of Figure 10 (1). Here, real objects with the same shape as that of the synthetic objects and shadows cast by those objects are shown in the top row for the comparison purpose.

In the proposed approach, since the entire illumination distribution of a scene is represented as a set of area light sources whose solid angle is adjusted depending on its stability, the distribution of shadows is a little different from those of real shadows cast by the real objects. However, it is found through our experiments that if we instead used a set of point light sources and examined only occlusions of the center points of those light sources, the entire estimation became unstable unless the method happened to find the correct locations for those three light sources with coarse sampling.

We have also tested our proposed method by using real images taken in an ordinary office environment. In the bottom row of Figure 10 (2), several synthetic objects were also superimposed onto the surface using the illumination distribution estimated from the input *shadow image* shown in this figure. It is worth noting that in this example, a relatively large area of the *shadow surface* is occluded by the *occluding object*, and it is often difficult to provide a correct estimate of the illumination distribution in such case. Even in this challenging case, our proposed approach could reliably estimate the illumination distribution of the scene by

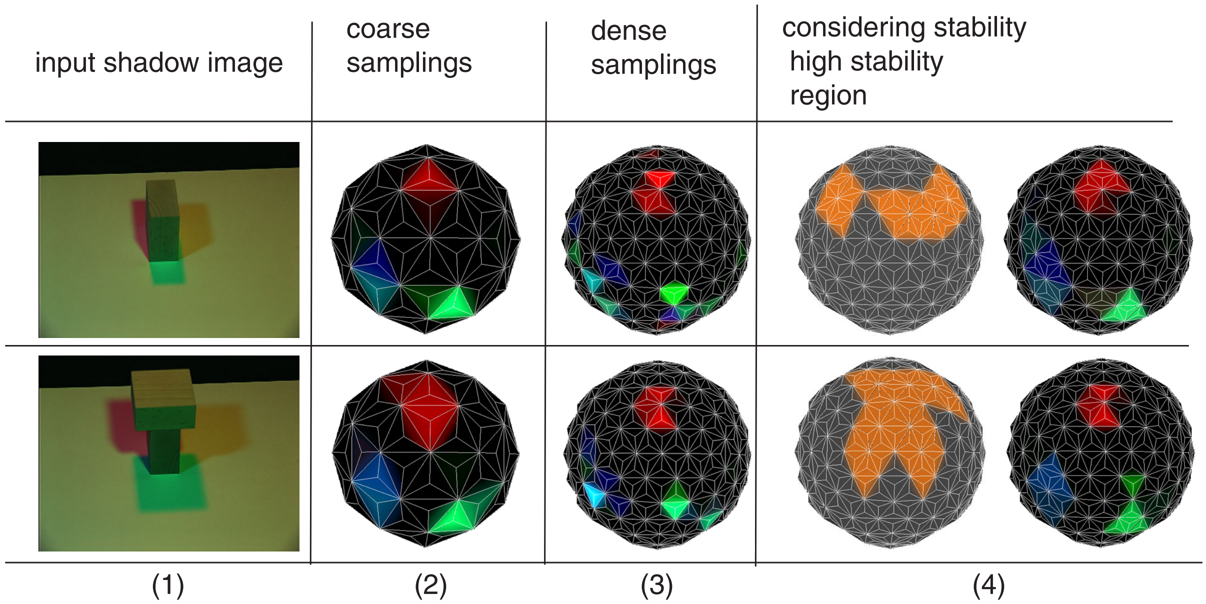

Figure 9: Illumination estimation without and with considering stability

taking stability issues into considerations. Shadows cast by those synthetic objects resemble well those cast by the real objects, and this shows that the estimated illumination distribution gives a good presentation of that of the real scene.

6 Conclusions

In this paper, we have proposed a method for estimating illumination distribution of a real scene from shadows in a given image in a reliable manner. First, we examined the source of instability of our previously proposed method [20] carefully in several aspects: invisible regions of shadows; limited sampling resolution for radiance distribution inside shadows; and approximation of illumination distribution as a collection of point light sources. Then, based on that analysis, we proposed methods to overcome those problems regarding the instability of the illumination estimation from shadows. For estimating the illumination distribution of a scene reliably, we adopted a strategy for changing the sampling density of the illumination distribution depending on the amount of the information obtainable from a *shadow image* for a particular direction of the illumination distribution. For using radiance distribution inside penumbra of shadows correctly, we introduced a super-sampling scheme for examining occlusion of incoming light from each light source. We also explained the optimal sampling of image pixels and the selection of illumination distribution samplings for more stable computation. All of these extensions contribute to improve stability and accuracy of illumination estimation from shadows. Unlike the previously proposed method for illumination estimation, illumination distribution can be estimated in a reliable manner with these proposed improvements regardless of types of input images such as the shape of an occluding object or a camera position.

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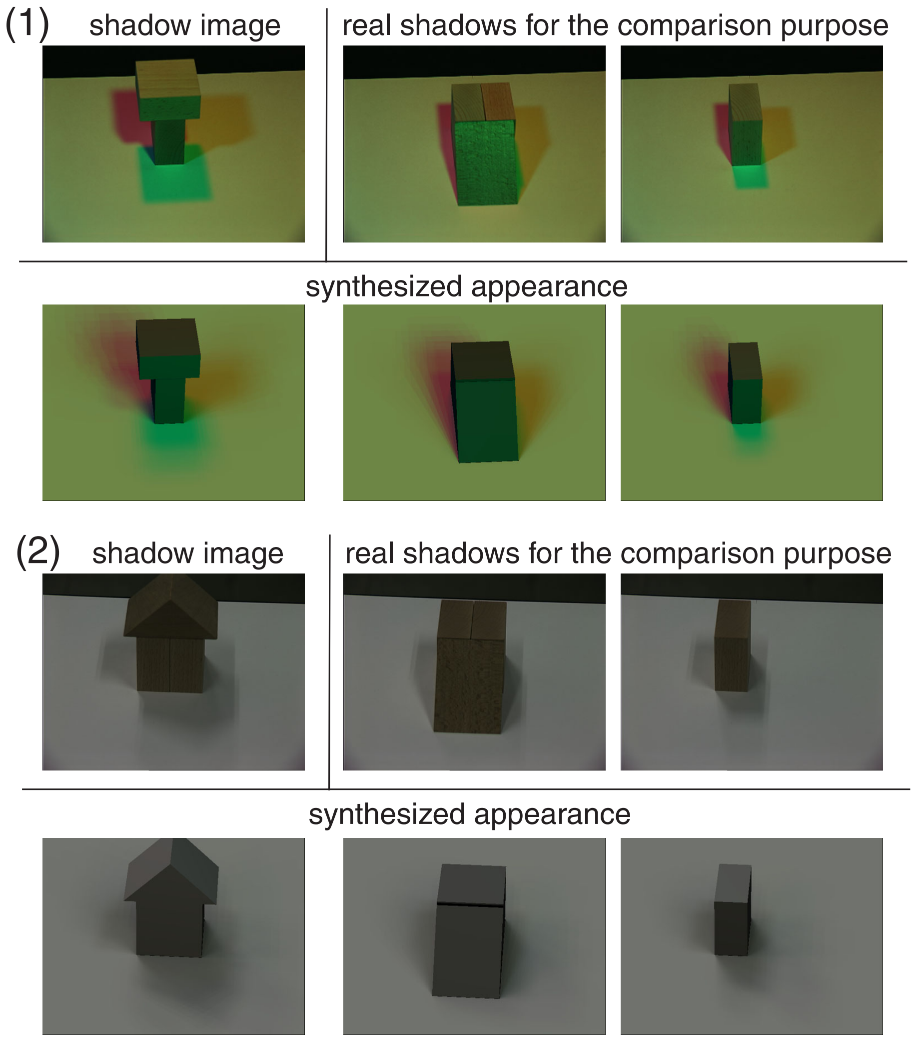

Figure 10: Synthesized appearance using the estimated illumination distribution

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