Our Experience of Graph Golf Competition

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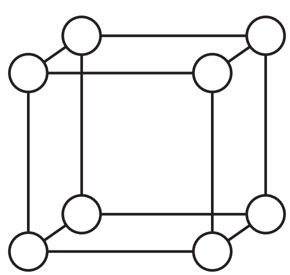
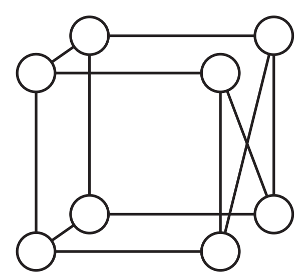
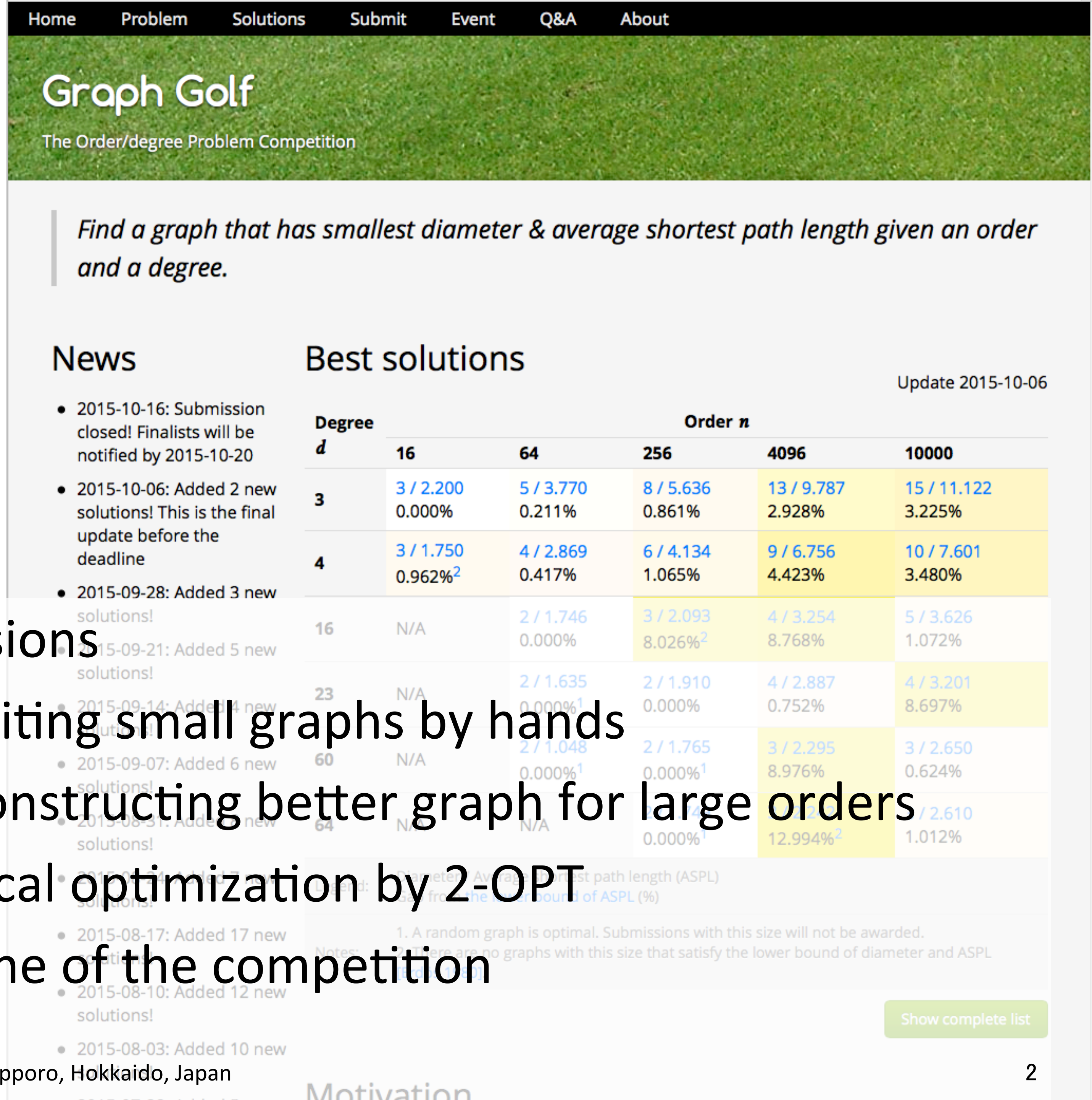


Cube; k=3, l=1.71 (Gap 9.1%)



Möbius loop; k=2, l=1.57 (Gap 0.0%)

Outline



Degree d	Order n: 16	64	256	4096	10000
3	3 / 2.200 0.000%	5 / 3.770 0.211%	8 / 5.636 0.861%	13 / 9.787 2.928%	15 / 11.122 3.225%
4	3 / 1.750 0.962%[2]	4 / 2.869 0.417%	6 / 4.134 1.065%	9 / 6.756 4.423%	10 / 7.601 3.480%
16	N/A	2 / 1.746 0.000%	3 / 2.093 8.026%[2]	4 / 3.254 8.768%	5 / 3.626 1.072%
23	N/A	2 / 1.635 0.000%[1]	2 / 1.910 0.000%	4 / 2.887 0.752%	4 / 3.201 8.697%
60	N/A	2 / 1.048 0.000%[1]	2 / 1.765 0.000%[1]	3 / 2.295 8.976%	3 / 2.650 0.624%
64	N/A	N/A	[illegible] 0.000%[1]	[illegible] 12.994%[2]	[illegible] / 2.610 1.012%

- Our submissions
- 1st step: Writing small graphs by hands
- 2nd step: Constructing better graph for large orders
- 3rd step: Local optimization by 2-OPT
- After deadline of the competition

Our submissions
Average shortest path length (l)

degree d	Order n: 16	64	256	4096	10000
3	2.200 (July 29, *2) (= best known)				
4	1.750 (July 29, *2) (= best known)				
16	(N/A)		2.09752 (Oct. 15, *3) (> 2.09274 best known)	(3.25426873 best known)	(3.62562128 best known)
23				2.902032 (Oct. 15, *3) (> 2.88676 best known)	3.201133 (Oct. 15, *3) (> 3.200897 best known)
60				2.295265 (Oct. 15, *1) 2.295275 (Aug. 28)	(2.650399 best known)
64				2.242214 (Oct. 15, *1) 2.242228 (Sep. 4)	(2.610117 best known)

Diameter k (lower bound; best known)

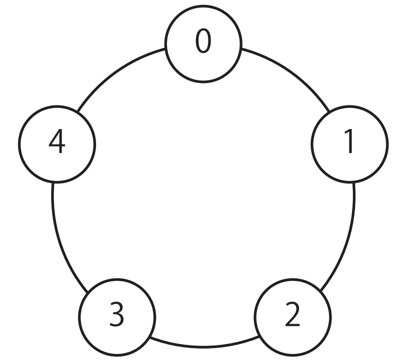
degree d	order n: 16	64	256	4096	10000
3	3; 3	5; 5	7; 8	11; 13	12;15
4	2; 3	4; 4	5; 6	7; 9	8; 10
16	N/A	2; 2	2; 3	4; 4	4; 5
23	N/A	2; 2	2; 2	3; 4	3; 4
60	N/A	2; 2	2; 2	3; 3	3; 3
64	N/A	N/A	2; 2	2; 3	3; 3

Note:
- *1: best solution?
- *2: best but late submission
- *3: our best but worse

1st step: Writing small graphs by hands (1/3)
Diameter and Cycles

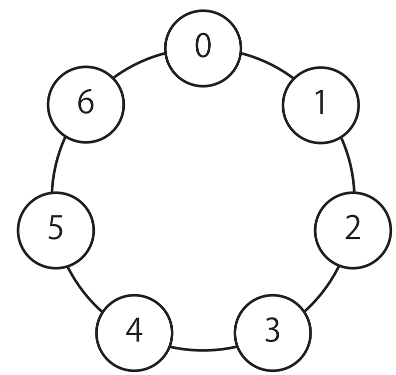
Diameter k = 3

- 5 cycle is the best, when degree d = 2
- 4 or less cycles are probably redundant, when degree d > 2



Diameter k = 4

- 7 cycle is the best, when degree d = 2
- 6 or less cycles are probably redundant, when degree d > 2

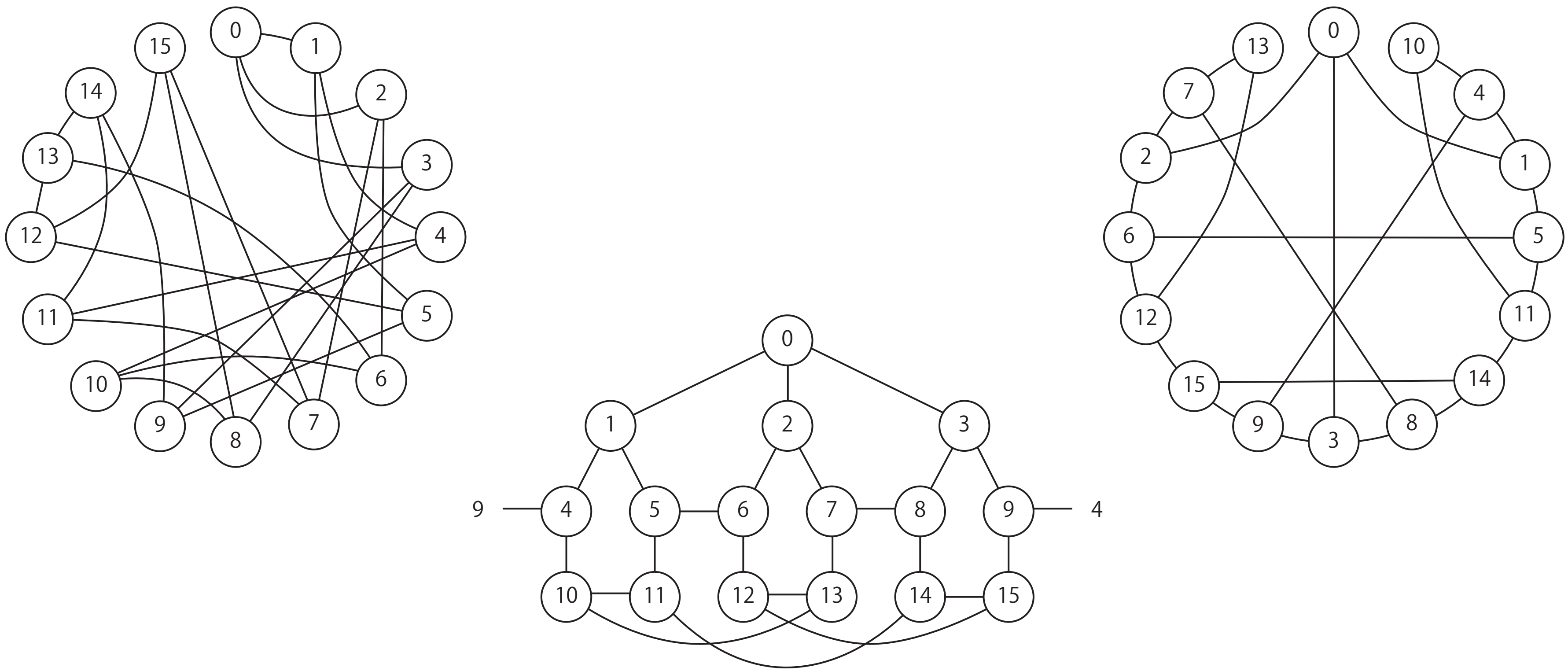


Note:

- Petersen graph: n=10, d=3; (3, 5)-cage. McGee graph: n=24, d=3; (3, 7)-cage
- After competition, we were found out the names of Petersen graph and McGee graph.

1st step: Writing small graphs by hands (2/3)

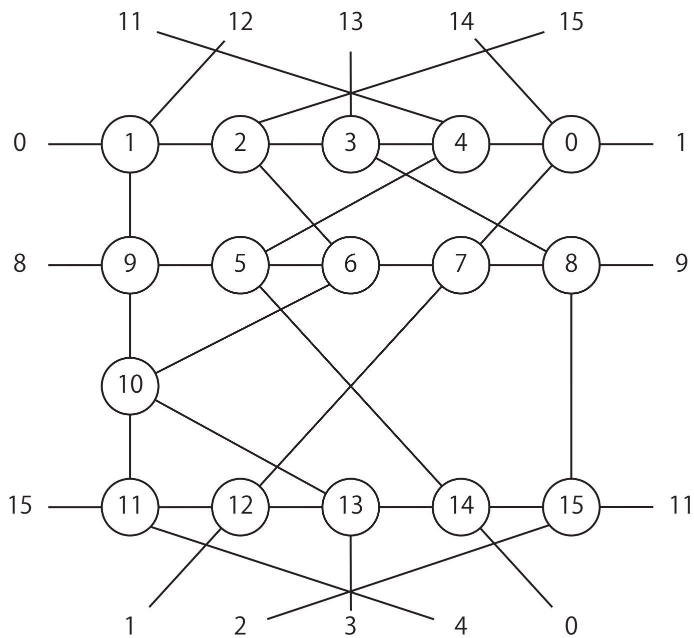
16 nodes, degree 3, k=3, l=2.200

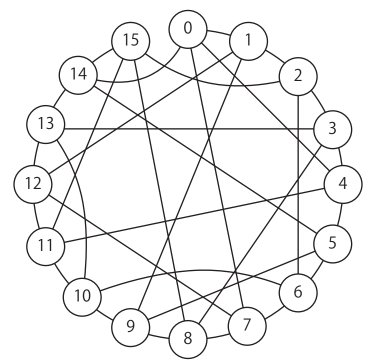


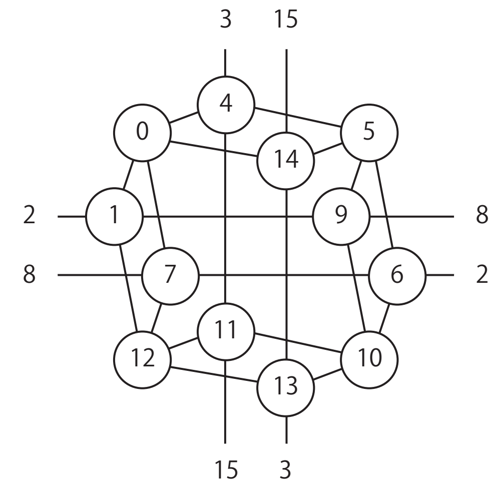
- No 4 cycles ... 4 cycles contains redundant path for k = 2 graph.

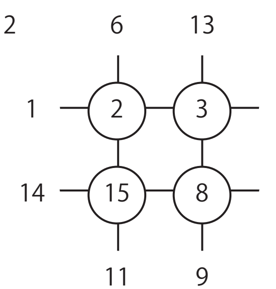
1st step: Writing small graphs by hands (3/3)

16 nodes, degree 4, k=3, l=1.750







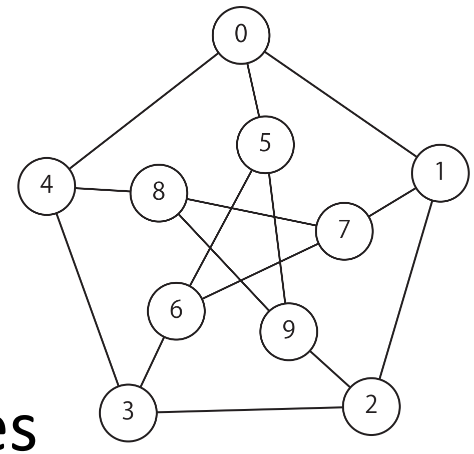


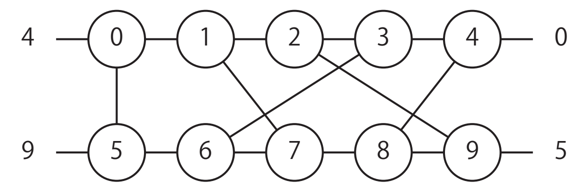
- # of 4 cycles: 5 (5-6-10-9, 10-11-12-13, 0-4-5-14, 0-1-12-7, 2-3-8-15)

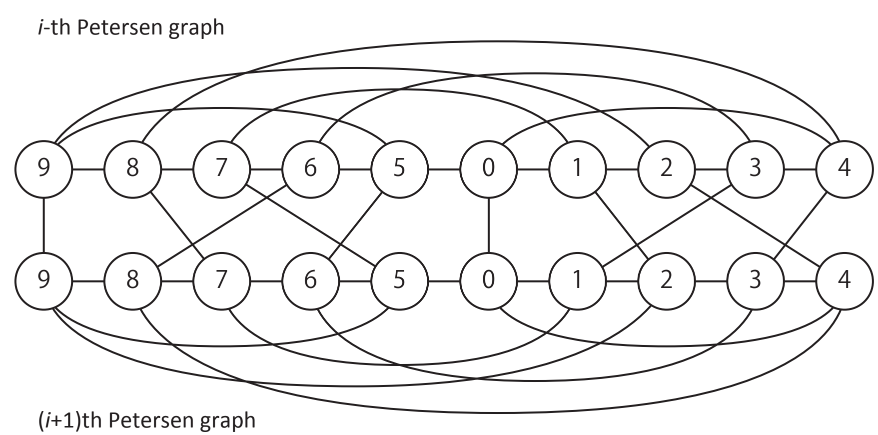
2nd step: Constructing better graph for larger cases (1/2)

Graphs of k=3

- Problems:
  - ✓n=256, d=16
  - ✓n=4096, d=60 and 64
  - ✓n=10000, d=60 and 64
- Algorithm: adding edges greedily, to increase # of 5 cycles
- Base graph: (n / 10) Petersen graphs are connected







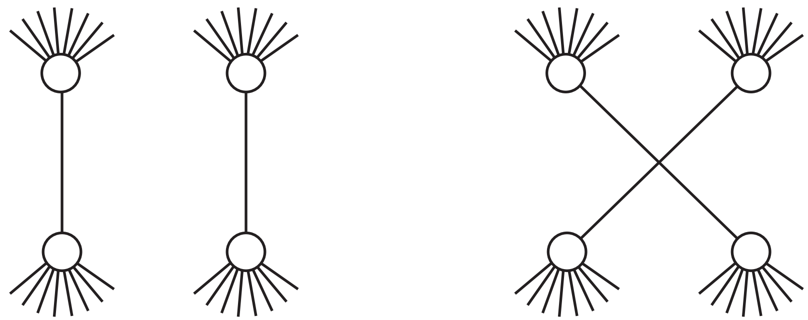
2nd step: Constructing better graph for larger cases (2/2)

Graphs of k=4

- Problems:
  - ✓ n=4096, d=23
  - ✓ n=10000, d=23
- Algorithm: adding edges to increase # of 7 cycles greedily
- Base graph: McGee graphs or line graph

3rd step: Local optimization of graphs (1/2)

2-OPT (flip two edges)



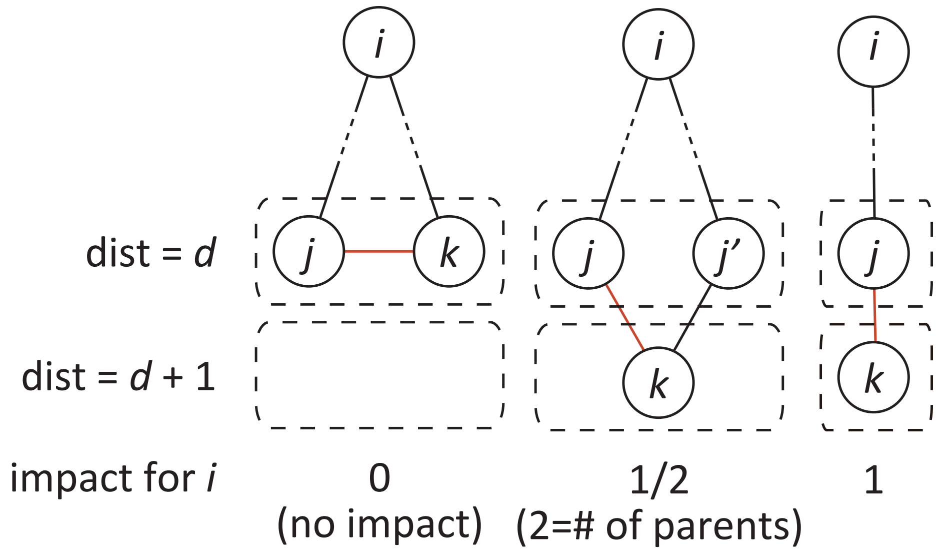
(e.g. degree 9)

- Keeping flipped graph, if flipped graph has better diameter / ASPL
- Question: which pair of edges is a better pair to shorten ASPL?
- An answer: edge impact or importance (see the next slide)

3rd step: Local optimization of graphs (2/2)

Edge Importance for fast 2-OPT

- Rough def.: an edge (*j*, *k*) has a positive value for node *i*, if *j* and *k* is <u>not</u> the same distance from *i*. Importance of edge (*j*, *k*) is the sum of such values for all node *i* ($i \neq j$ and *k*).



- Calculate importance for each edge (*j*, *k*), sort edges, and try flip of a pair of edges which has smaller importance.

After deadline of the competition

Table of our updates (ASPL l)

k (lower bound; best known)			
degree	**order n**		
d	**256**	**4096**	**10000**
16	2; 3	4; 4	4; 5
23	2; 2	3; 4	3; 4
60	2; 2	3; 3	3; 3
64	2; 2	2; 3	3; 3

degree	**Order n**		
d	**256**	**4096**	**10000**
16	2.09274 (2-OPT, Dec. 9, *3) 2.09751 (2-OPT, Oct. 15) (< 2.09262 best known)	3.286563 (Const., Oct. 1, *3) (> 3.252718 best known)	(3.625174 best known)
23	(1.90980 best, no gap)	2.900162 (2-OPT, Dec. 9, *3) 2.902032 (2-OPT, Oct. 15) (> 2.886137 best known)	3.201116 (2-OPT, Dec. 8, *3) 3.201121 (Const., Nov. 12) (> 3.200257 best known)
60	(1.76470 random, no gap)	2.295241 (2-OPT, Dec. 7, *1) 2.295265 (2-OPT, Oct. 15) 2.295275 (Const., Aug. 28)	2.648979 (2-OPT, Nov. 8 , *1) 2.648980 (Const., Nov. 2) (< 2.650157 best known)
64		2.242193 (2-OPT, Dec. 7, *1) 2.242214 (2-OPT, Oct. 15) 2.242228 (Const., Sep. 4)	2.611305 (2-OPT, Dec. 9, *3) 2.611310 (Const. Oct. 26, *3) (> 2.609927 best known)

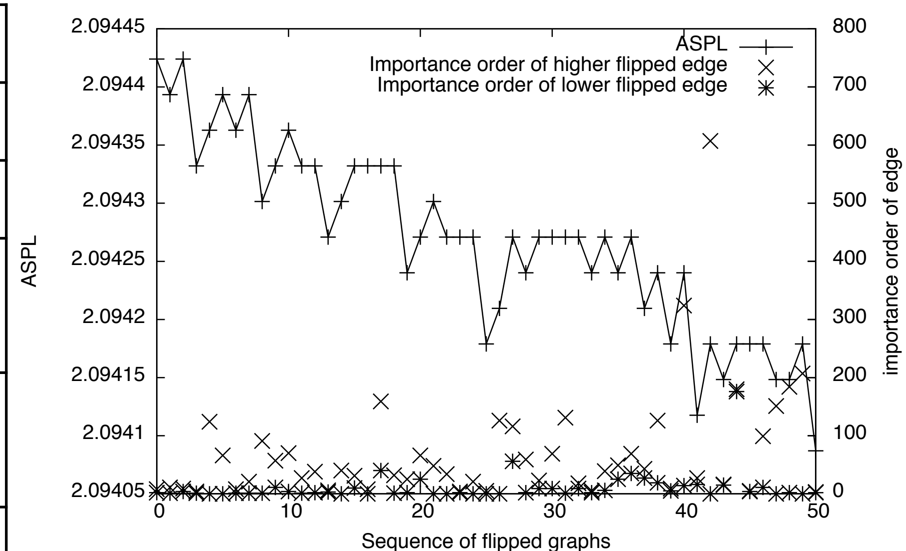
Note:
*1: best solution?
*3: our best but worse

Conclusion

- Fortunately, we found some smaller graphs than others.
- We fully enjoyed as same as our research, from the middle of July . (Some jobs might be suspended or delayed. It has been hard to stop thinking, even after deadline)
- Larger problems consumed long time of computers and ourselves.
- We used several PCs. (a small PC, Mac mini, and a department's lack-mount server)
- We are not so young, but tried, and tired.
- Latest graph at http://www.cs.kumamoto-u.ac.jp/~kitasuka/tmp/graphgolf-20151209.tgz

Appendix
Unit of ASPL (known experimentally)

Order *n*	**unit of ASPL**	**degree**
16	?	
64	?	
256	3.1*10^{-5} =0.00003063725490	*d*=16
4096	1.2*10^{-7} =0.00000011923840	*d*=23, 60, 64
10000	2.0*10^{-8} =0.00000002000200	*d*=23, 60, 64



An example 2-OPT sequence
(n=256, d=16, l=2.09424–2.09409)