The construction of a regular graph

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Graphs of Diameter=2 in Graph Golf

Degree d	Order n				
	16	64	256	4096	10000
3	3 / 2.200	5 / 3.770	8 / 5.636 [illegible]1%	13 / 9.787 2.928%	15 / 11.122 3.225%
4	[illegible] 0.962%[2]	[illegible] 0.4[illegible]%	6 / 4.134 1.065%	9 / 6.756 4.423%	10 / 7.601 3.480%
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64	N/A	N/A	[illegible] 0.000%[1]	[illegible] 12.994%[2]	3 / 2.610 1.012%

(Order,Degree)=(64,16)

(Order,Degree)=(256,23)

Outline

(1) How to make a regular graph

(2) Graphs of Diameter=2 in Graph Golf

(3) Summary

How to make a regular graph

(A) Making a regular graph directly

(B) Making a regular graph from small regular graphs

Constructing methods (Diameter=2)

(A) (B)

	Brown's Construction	Generalized Brown's Construction	Copying pentagon	Copying G_8	Strong product
Order	$p^{2k}+p^k+1$	Ex. $p^{2k}+p^{2k-1}+p^{2k-2}$	5k	8k	n,m ↓ nm
Degree	p^k+1	p^k+p^{k-1}	k+1	k+2	d_1,d_2 ↓ $d_1d_2+d_1+d_2$
Order/Degree^2	~ 1	~ 1	~ 5/k	~ 8/k	

(p : a prime k : a natural number)

Constructing methods (Diameter=2)

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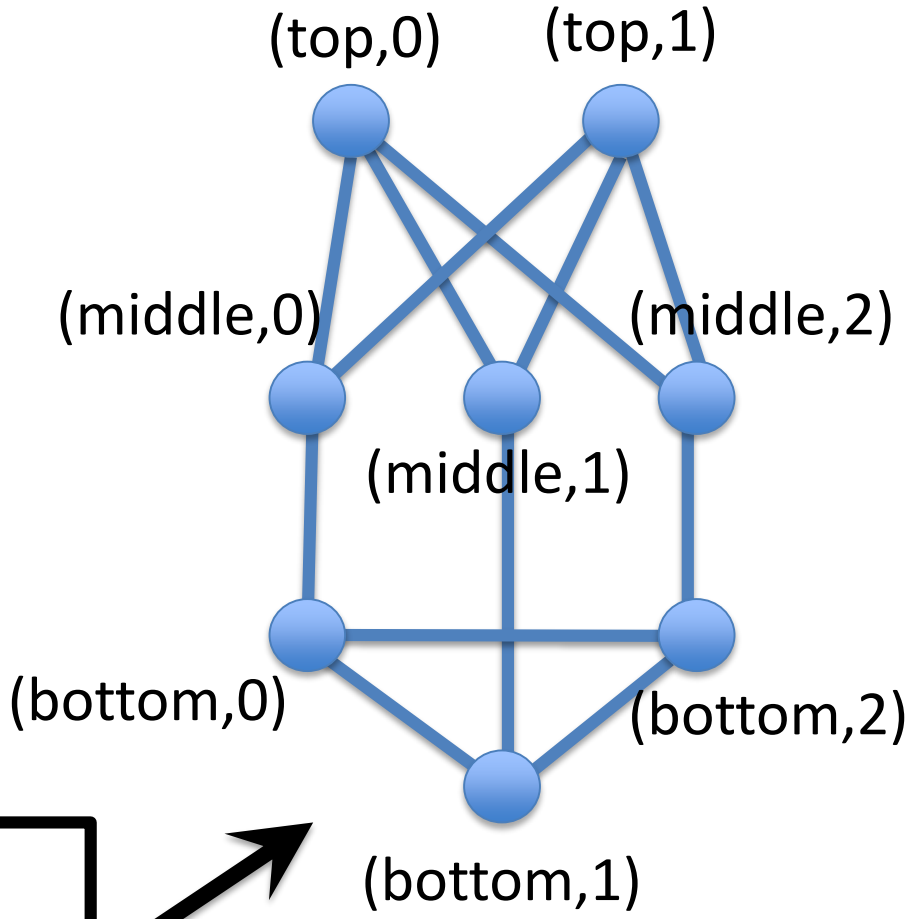
Constructing method : Copying G_8

$G_8 \equiv$

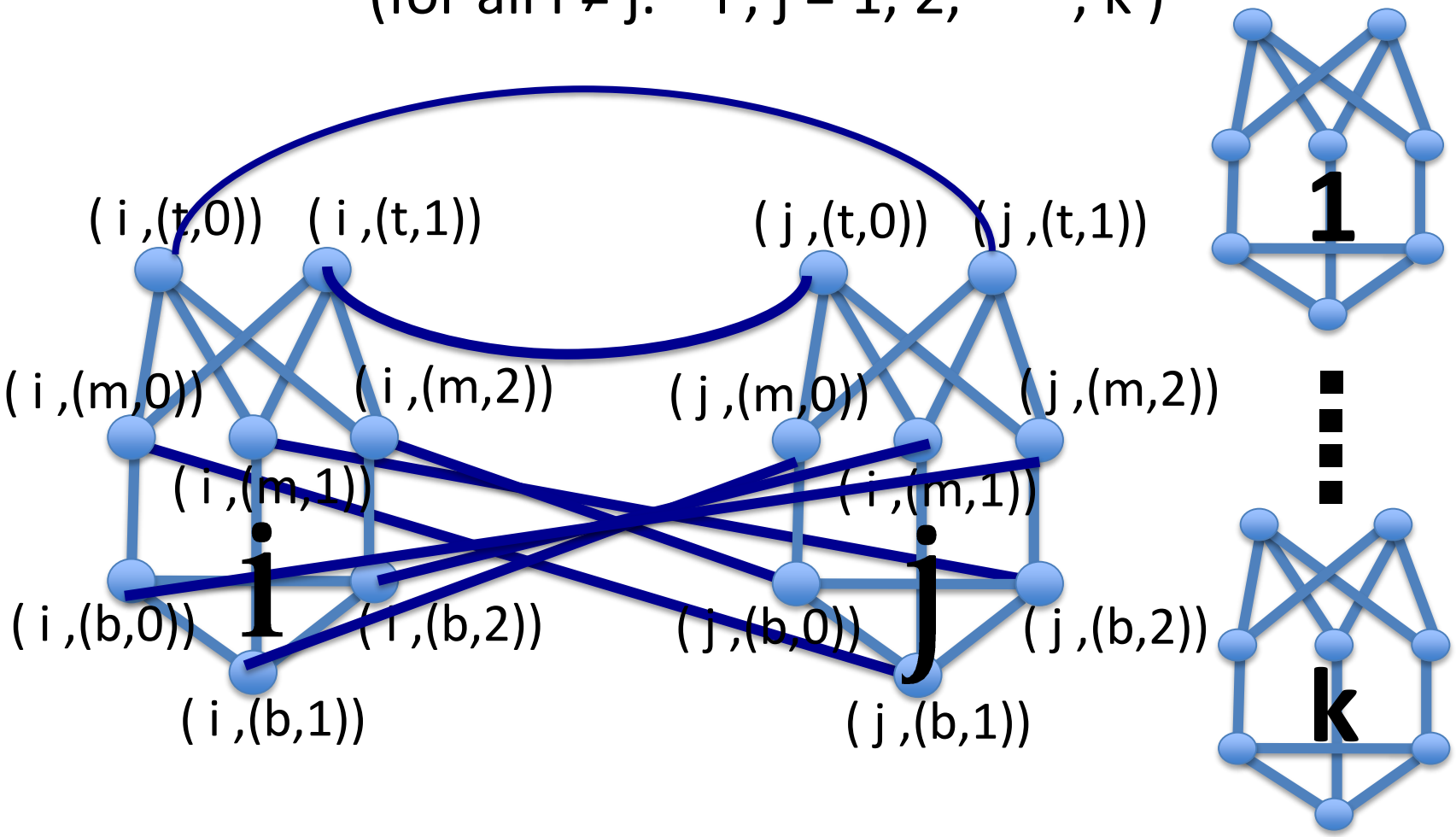


A regular graph of order=8, degree=3 and diameter=2

Constructing method : Copying G_8

Connect k-copies of G_8 as follows:

(for all i ≠ j. i , j = 1, 2, ･･･, k)



✔**This Graph has 8k nodes** ✔**degree=k+2**

Constructing methods (Diameter=2)

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(1) How to make a regular graph

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(3) Summary

Graphs of Diameter=2 in Graph Golf

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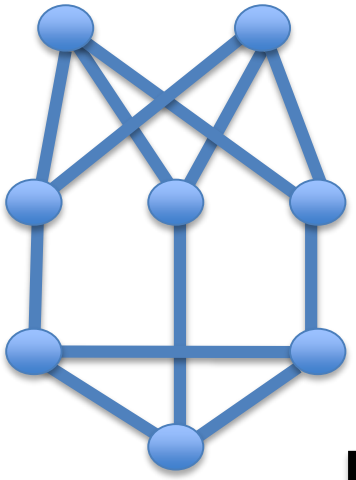
(Order,Degree)=(64,16)

How to make the (64,16) graph

Connecting 8-copies of G_8

Order=8
Degree=3



× 8

=

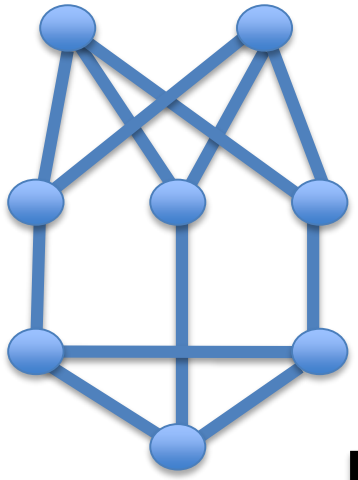
The graph of

Order = 8*8 = 64

Degree = 8 + 2 = 10

How to make the (64,16) graph

Connecting 8-copies of G_8

$G_{8\times 8} \equiv$  $\times\ 8$

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Graphs of Diameter=2 in Graph Golf

Degree d	Order n				
	16	**64**	**256**	**4096**	**10000**
3	3 / 2.200 0.000%	5 / 3.770 0.211%	8 / 5.636 0.861%	13 / 9.787 2.928%	15 / 11.122 3.225%
4	3 / 1.750 0.962%[2]	4 / 2.869 0.417%	6 / 4.134 1.065%	9 / 6.756 4.423%	10 / 7.601 3.480%
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(Order,Degree)=(256,23)

How to make the (256,23) graph

We can not use the strong product or copying G_8 method directly in this case

Because…

(1) The graph obtained from using 32-copies of G_8 is a graph of order = 256 and degree 34

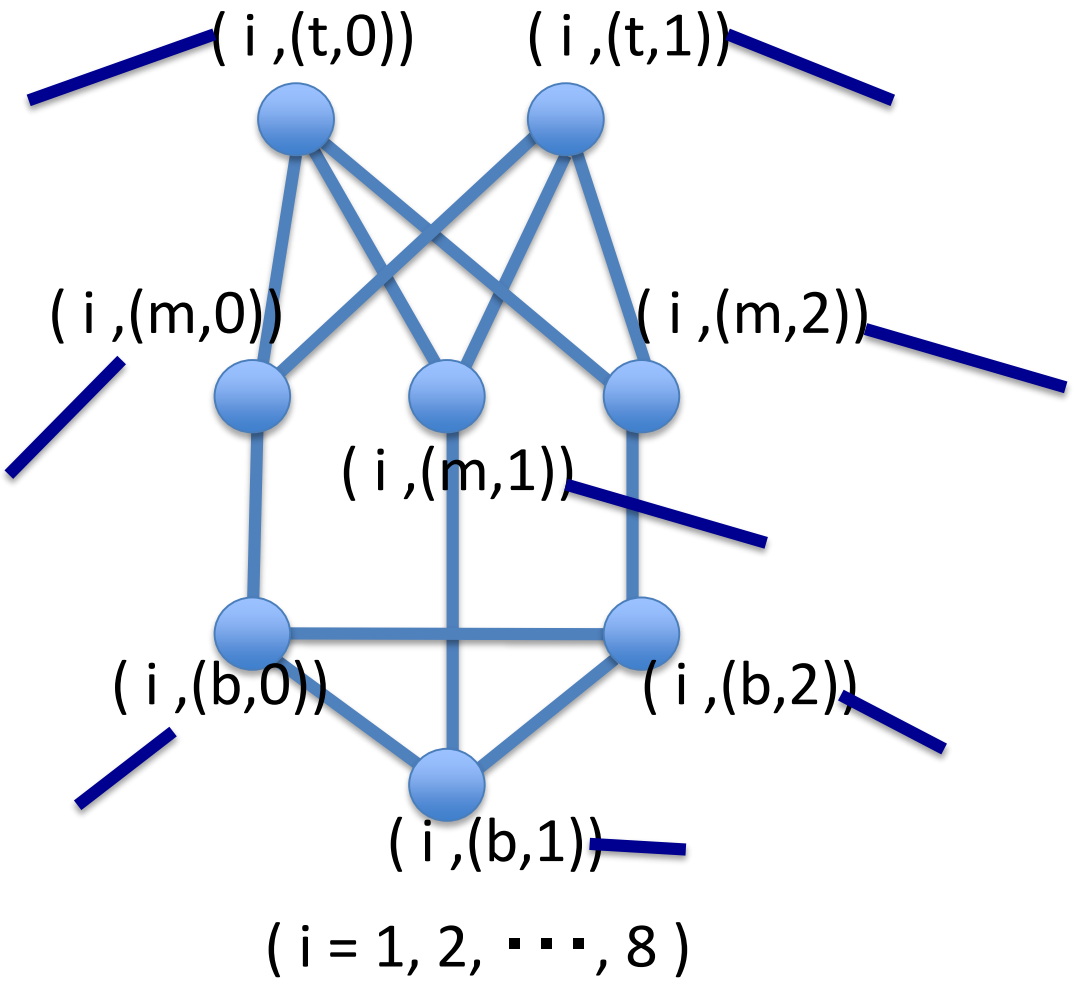
(1) There is no graph pair that their strong product satisfies Order = 256 and Degree $\leqq$ 23

We developed another method

How to make the (256,23) graph

New method : Copying $G_{8\times 8}$

$G_{8\times 8} =$

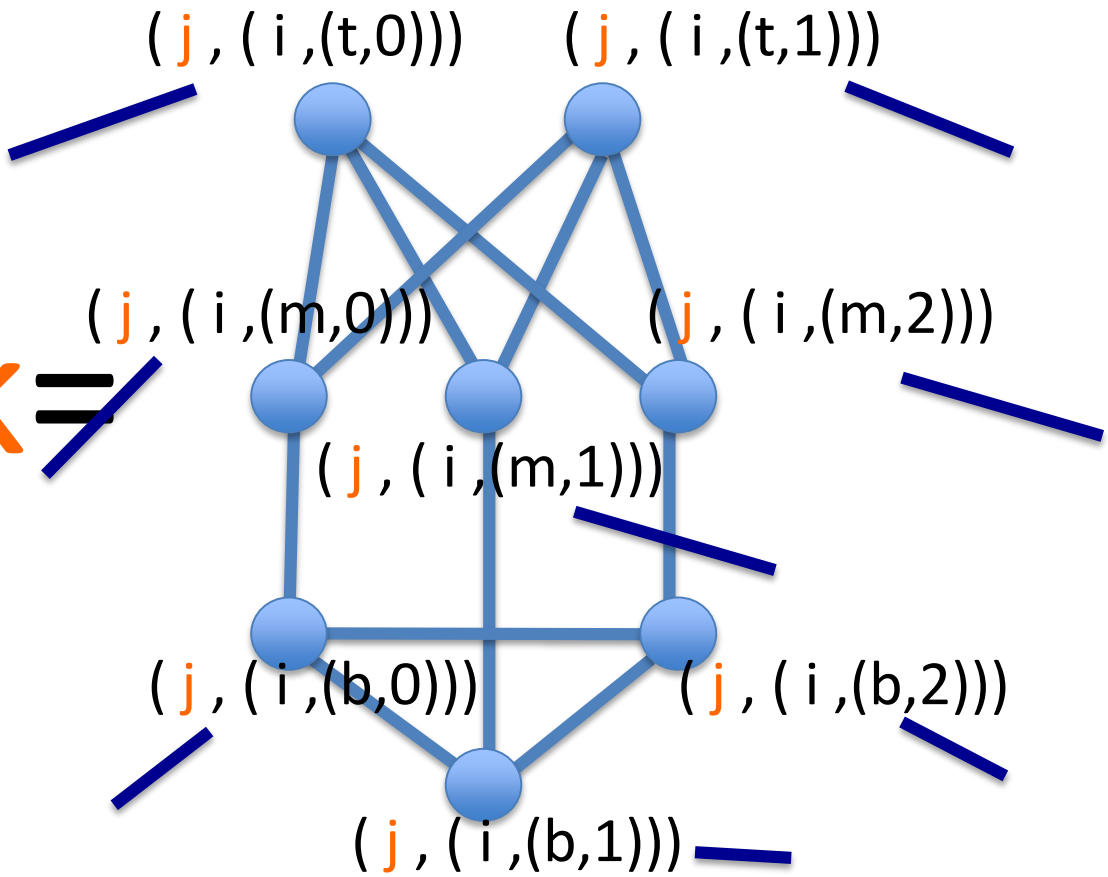


(i = 1, 2, ⋯, 8)

How to make the (256,23) graph

New method : Copying $G_{8\times 8}$

$G_{8\times 8} \times k \equiv$



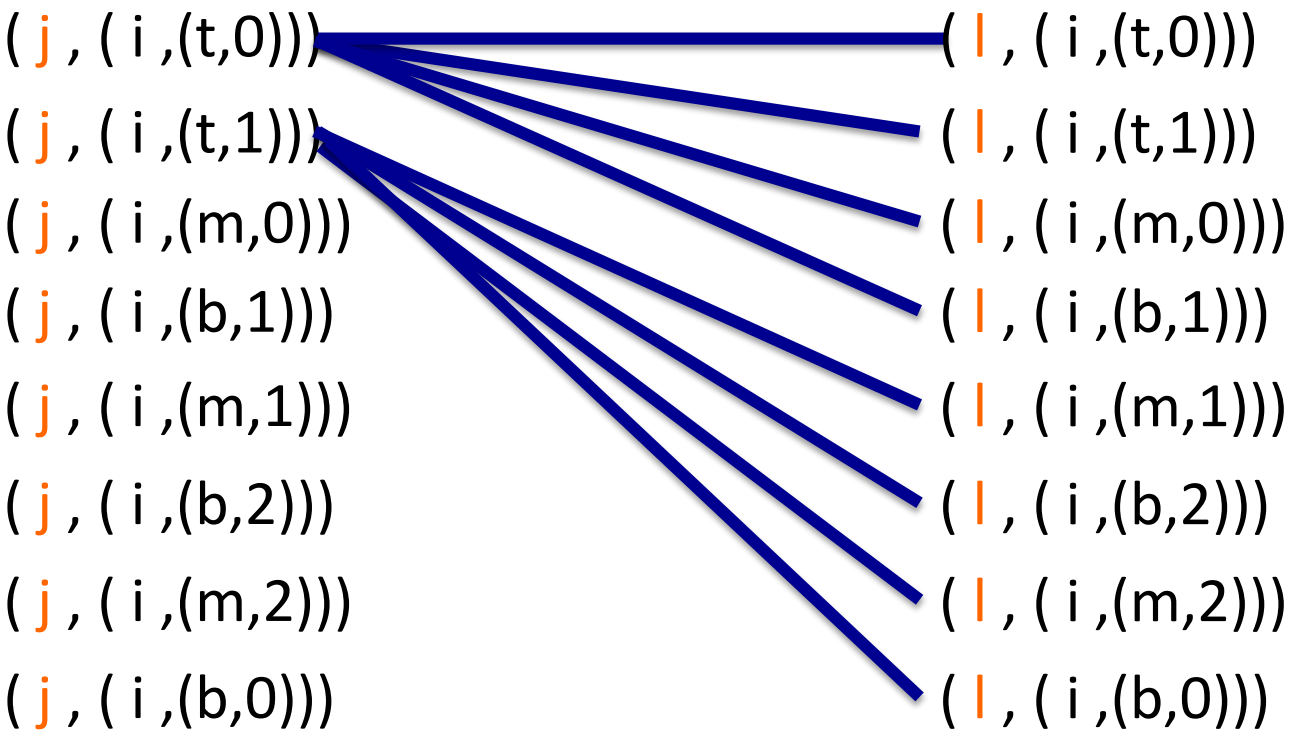
(j = 1, 2, ⋯, k i = 1, 2, ⋯, 8)

How to make the (256,23) graph

New method : Copying $G_{8\times 8}$

Connect k copies of $G_{8\times 8}$ as follows

(for all j > l. j , l = 1, 2, ▪ ▪ ▪, k)



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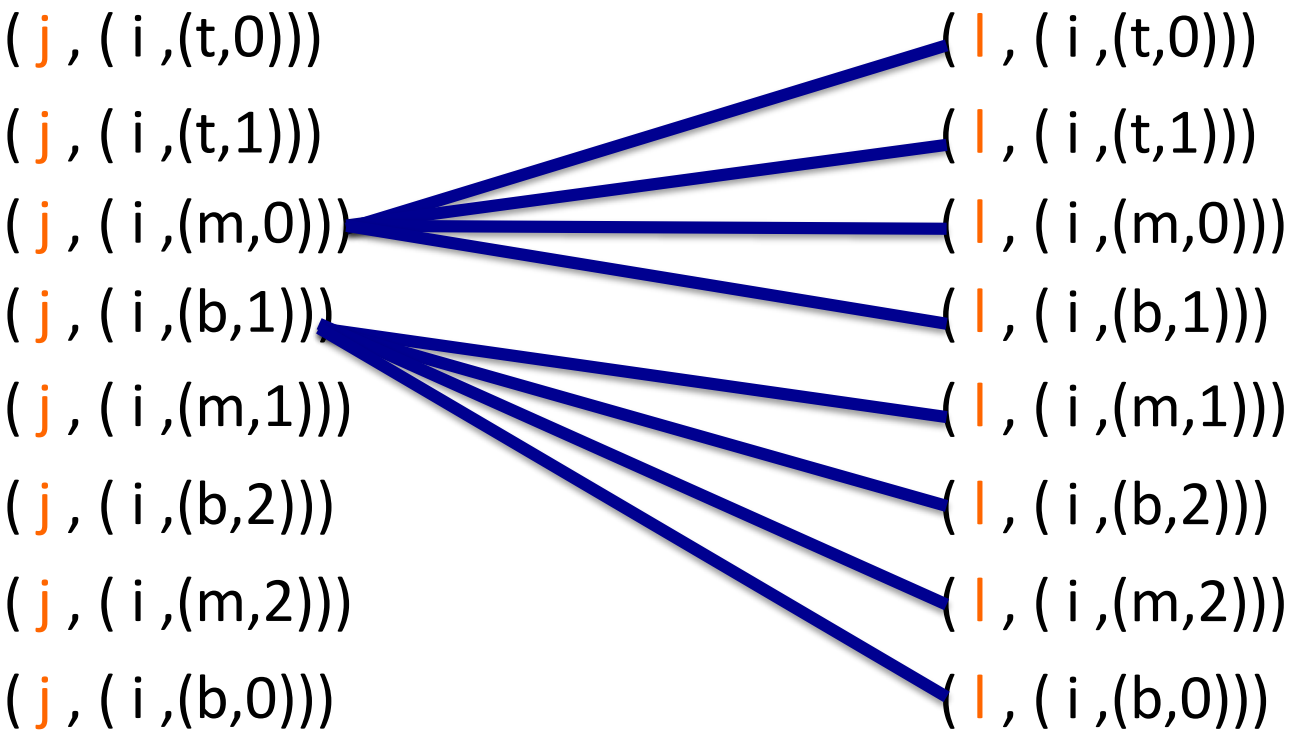

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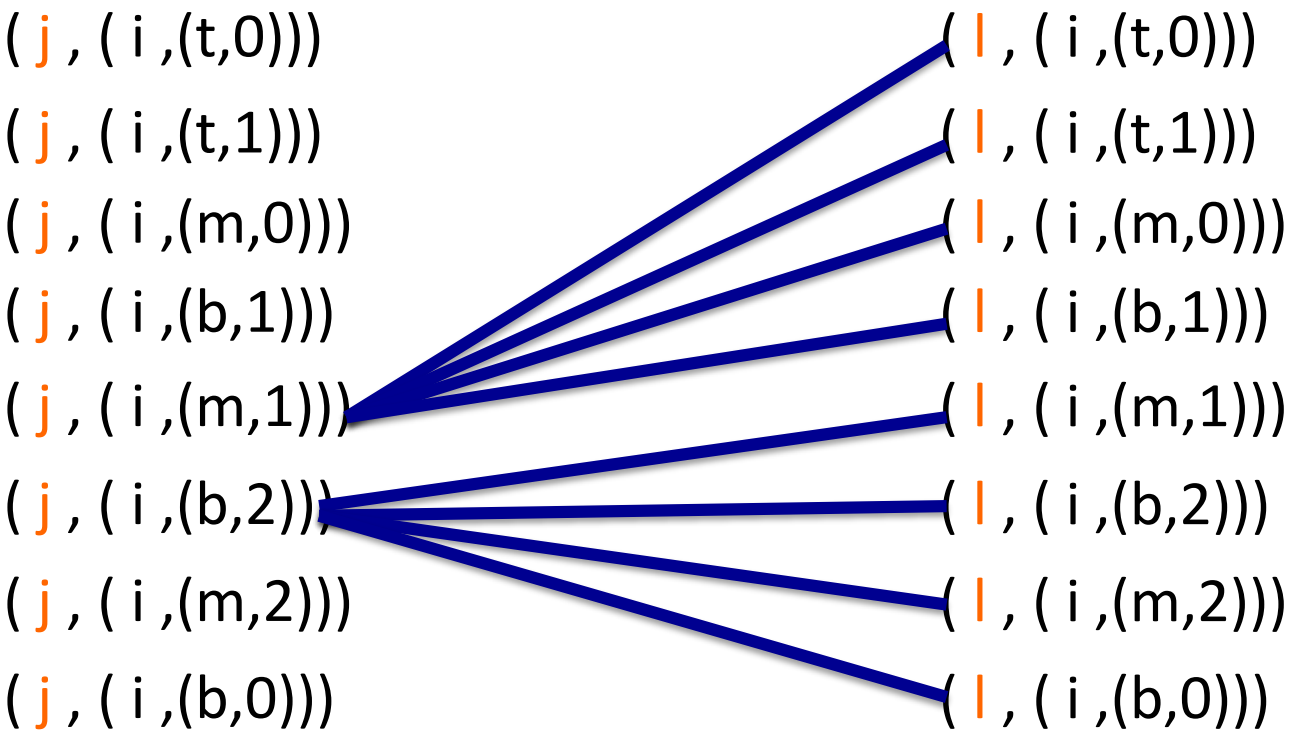

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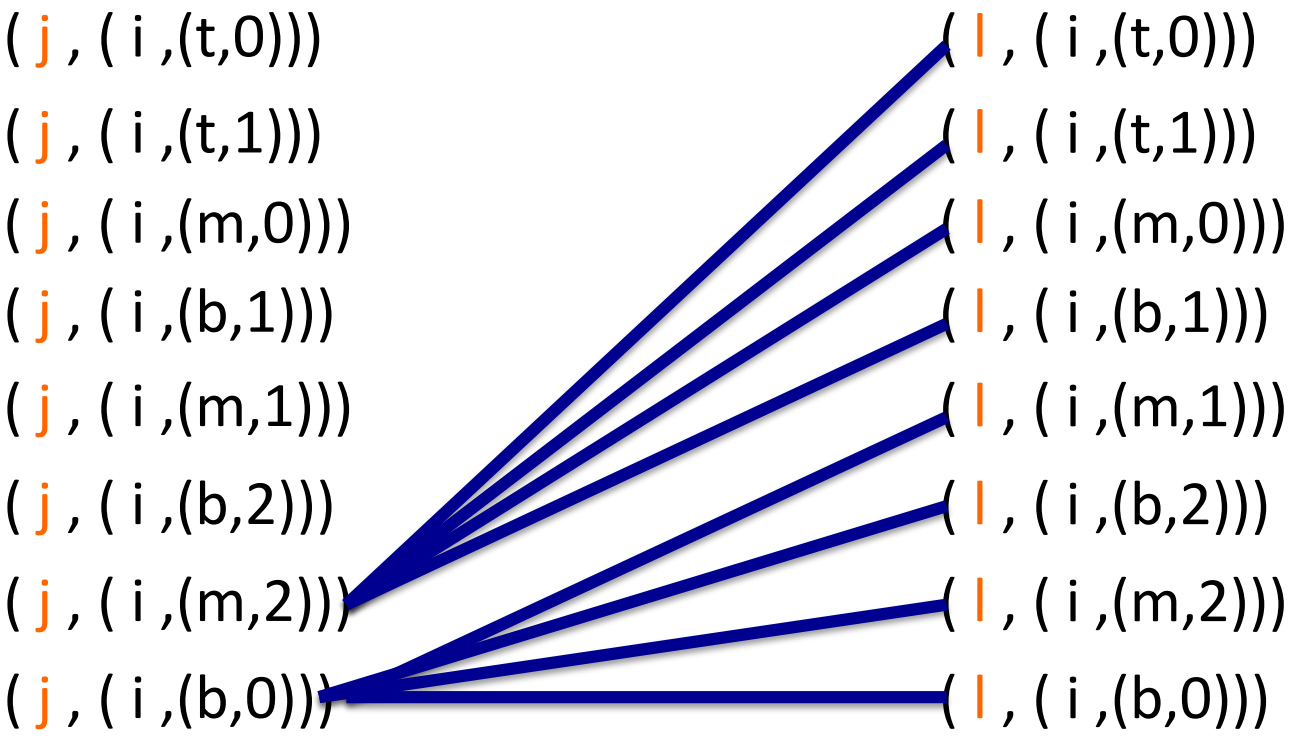

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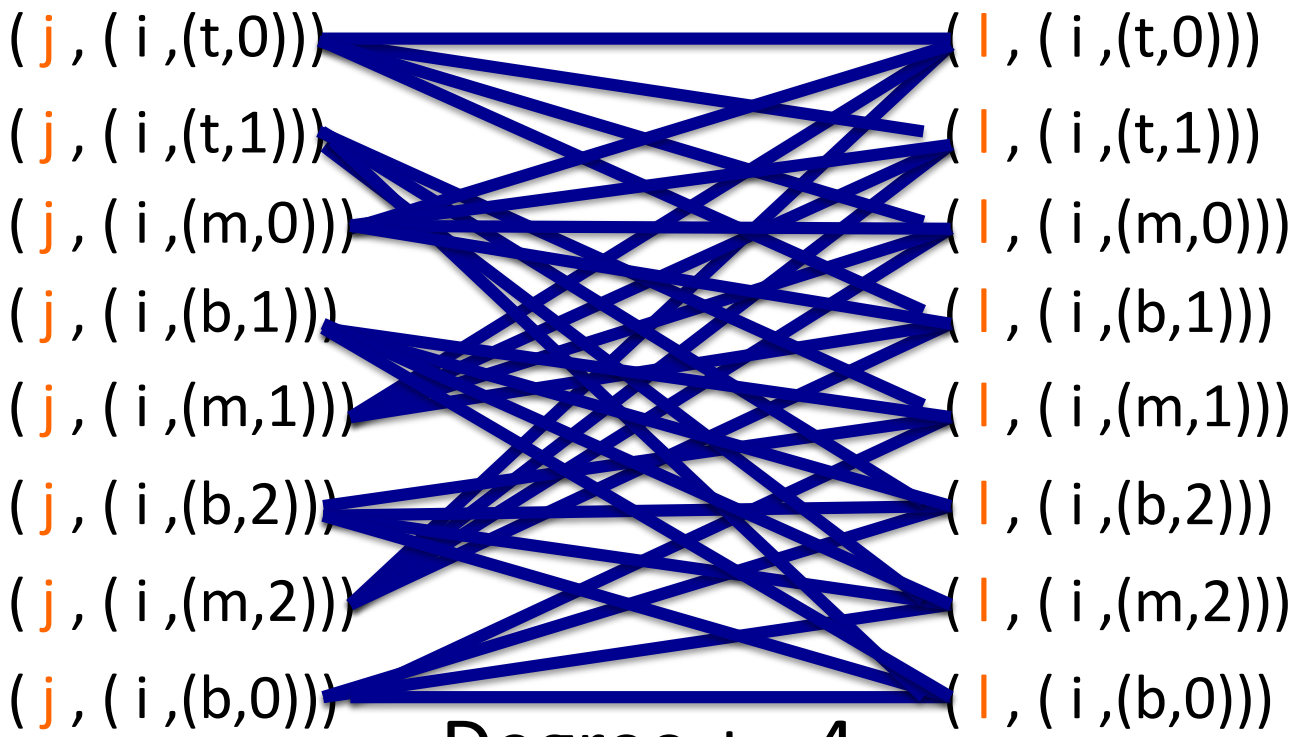

How to make the (256,23) graph

	Copying $G_{8\times 8}$	Copying G_8
Order	64k	8k
Degree	4k + 6	k+2
Order/Degree^2	~ 256/Order	~ 64/Order

How to make the (256,23) graph

	Copying $G_{8\times 8}$	Copying G_8
Order	$64*4 = 256$	8k
Degree	$4*4 + 6 = 22$	k+2
Order/Degree^2	~ 256/Order	~ 64/Order

Outline

(1) How to make a regular graph

(2) Graphs of Diameter=2 in Graph Golf

(3) Summary

Summary

- We made new constructing method of graph of diameter = 2.

- There are many graph constructing method.

They give some bounds for Order/Diameter
Degree/Diameter problem.
Order/Degree

Strong product of graphs

The strong product $G \boxtimes H$ of graphs G and H is a graph such that

the vertex set of $G \boxtimes H$ is the Cartesian product $V(G) \times V(H)$; and

any two distinct vertices (u,u') and (v,v') are adjacent in $G \times H$ if and only if:

- u is adjacent to v and u'=v', or
- u=v and u' is adjacent to v', or
- u is adjacent to v and u' is adjacent to v'

✔Strong product conserves diameter

✔Order($G \boxtimes H$)=Order(G) * Order(H)

✔Degree($G \boxtimes H$)=Degree(G)Degree(H)+Degree(G)+Degree(H)

Constructing methods (Diameter=2)

(A) (B)

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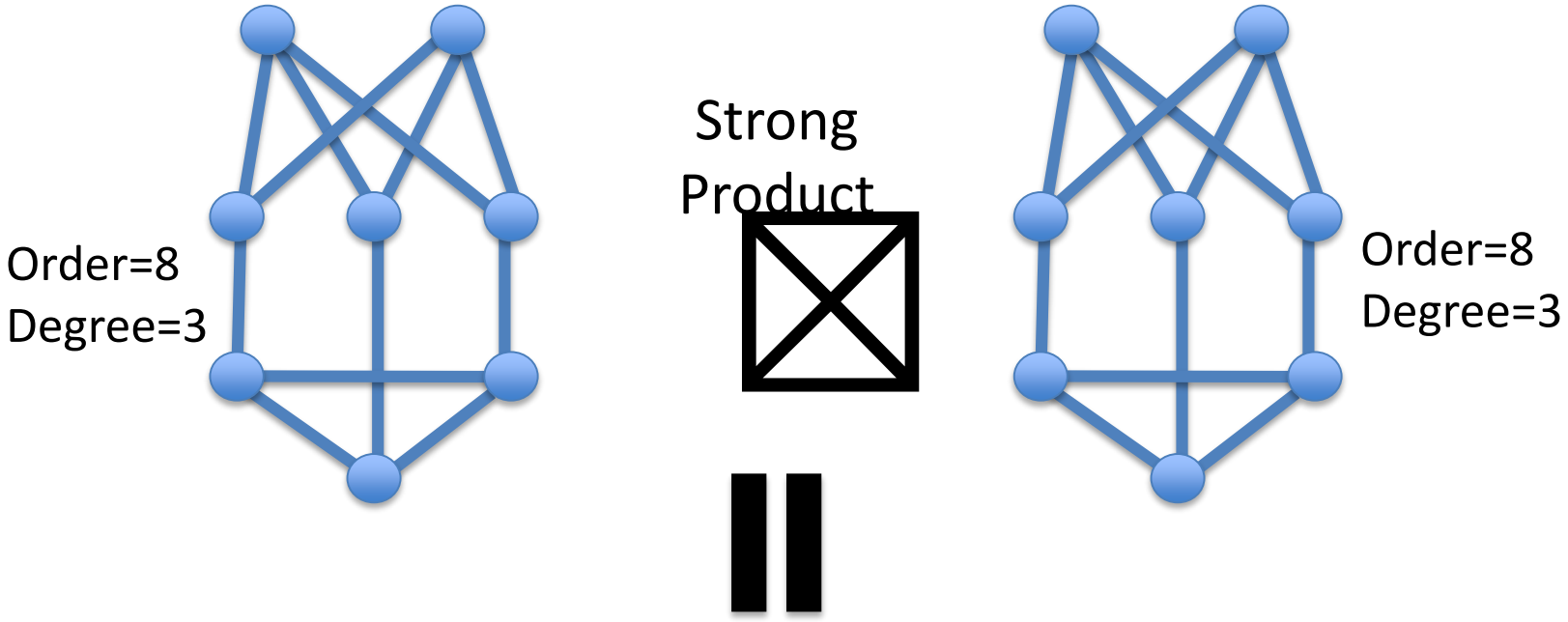
(p : a prime k : a natural number)

How to make the (64,16) graph

Method 1



The graph of

Order = 8*8 = 64

Degree = 3*3 + 3 + 3 = 15