Workshop in CANDAR'17, Aomori, Japan 2017-11-22

Graph Golf 2017

The order/degree problem competition

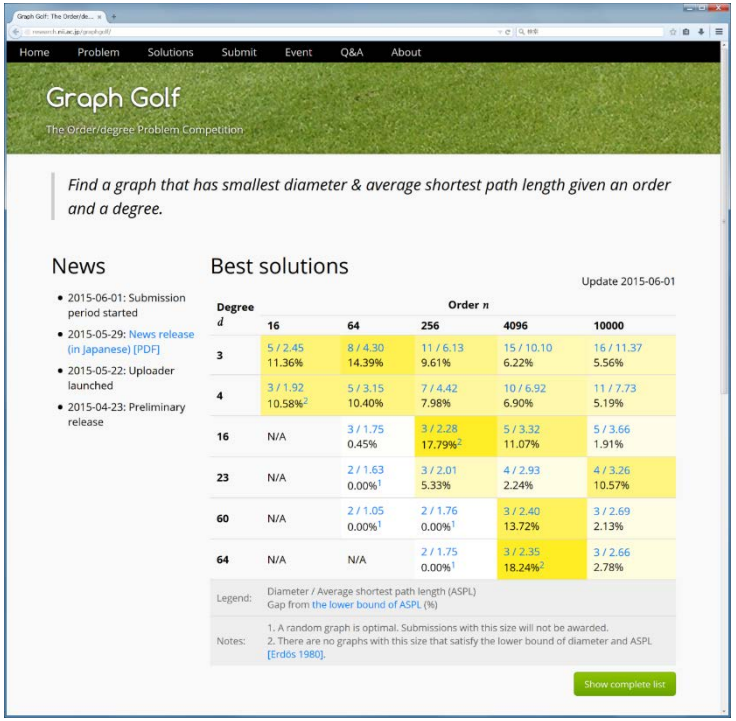
Timetable

- 13:15–13:25 Intro & Award Ceremony
- 13:25–14:15 Keynote
- 14:15–14:30 Break
- 14:30–15:00 Talk 1
- 15:00–15:30 Talk 2
- 15:30–15:45 Discussion

Introduction

Graph Golf

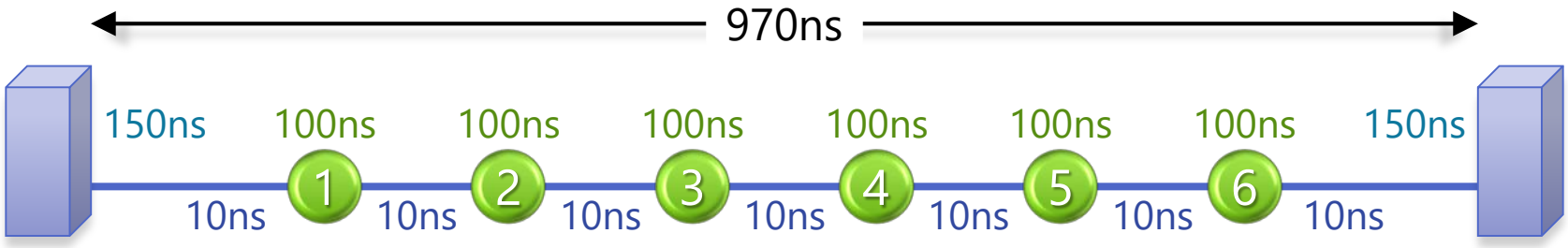
- An online competition for the order/degree problem since 2015
- Aims at building a catalog of “best” graphs for each order/degree pair



Winners of the 2015 competition

Motivation

- Low-latency networks are desired in supercomputers
  - 1μs for 3M cores ≒ **7 hops for 100K nodes**



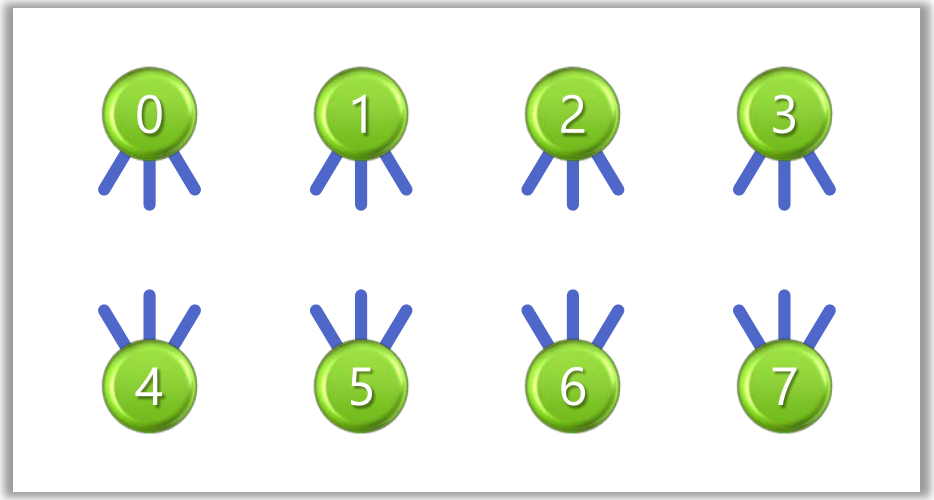
- Randomized topologies seem promising
  - Really? The best topology is unique, isn't it?
  - Diameters of random graphs are larger than the theoretical lower bound
  - Well, let's seek **the best topology!**

Order/Degree Problem (ODP)

- Given an order n and a degree d, find the graph with the smallest diameter k.
  - Among those with the same diameter, find the one with the smallest ASPL l.

A straightforward solution to design interconnects

- Practice: $n = 8$, $d = 3$



Moore Bound

- The lower bound $K_{n,d}$ of diameter k for order n and degree d

$$K_{n,d} = \begin{cases} \left\lceil \frac{n-1}{2} \right\rceil & \text{if } d = 2 \\ \left\lceil \log_{d-1}\left(\frac{(n-1)(d-2)}{d} + 1\right) \right\rceil & \text{if } d > 2 \end{cases}$$

- The lower bound $L_{n,d}$ of ASPL l for order n and degree d

$$L_{n,d} = \begin{cases} 1 & \text{if } K_{n,d} = 1 \\ \frac{\sum_{i=1}^{K_{n,d}-1} id(d-1)^{i-1} + K_{n,d}\left(n-1-\sum_{i=1}^{K_{n,d}-1} d(d-1)^{i-1}\right)}{n-1} & \text{if } K_{n,d} \geq 2 \end{cases}$$

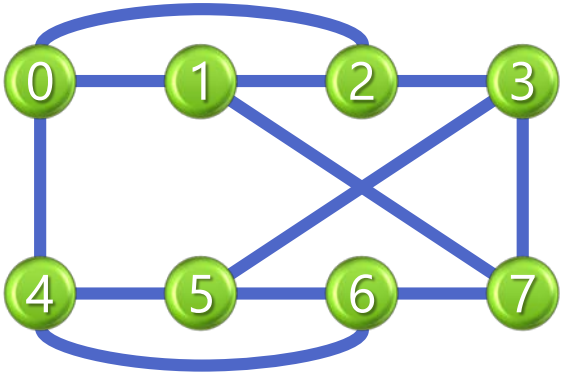
- ASPL gap p

$$p = \frac{l - L_{n,d}}{L_{n,d}}$$

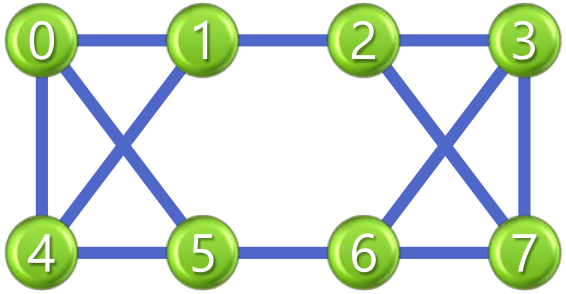
Practice - Answers

Lower bound of ASPL $L_{n,d} = 1.571$

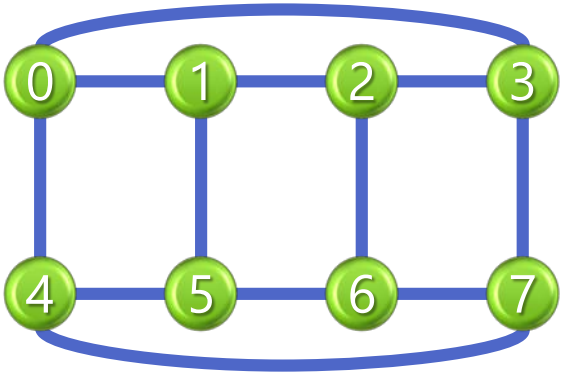
$k = 2$
$l = 1.571$
$p = 0\%$



$k = 3$
$l = 1.643$
$p = 4.55\%$



$k = 3$
$l = 1.786$
$p = 13.6\%$



$k = 3$
$l = 1.714$
$p = 9.09\%$

Award Rules

Widest Improvement Award

- Who find the largest number of "best" solutions
- The best solution means a graph with the smallest diameter, and with the smallest ASPL among those with the same diameter, for each order/degree pair.

Deepest Improvement Award

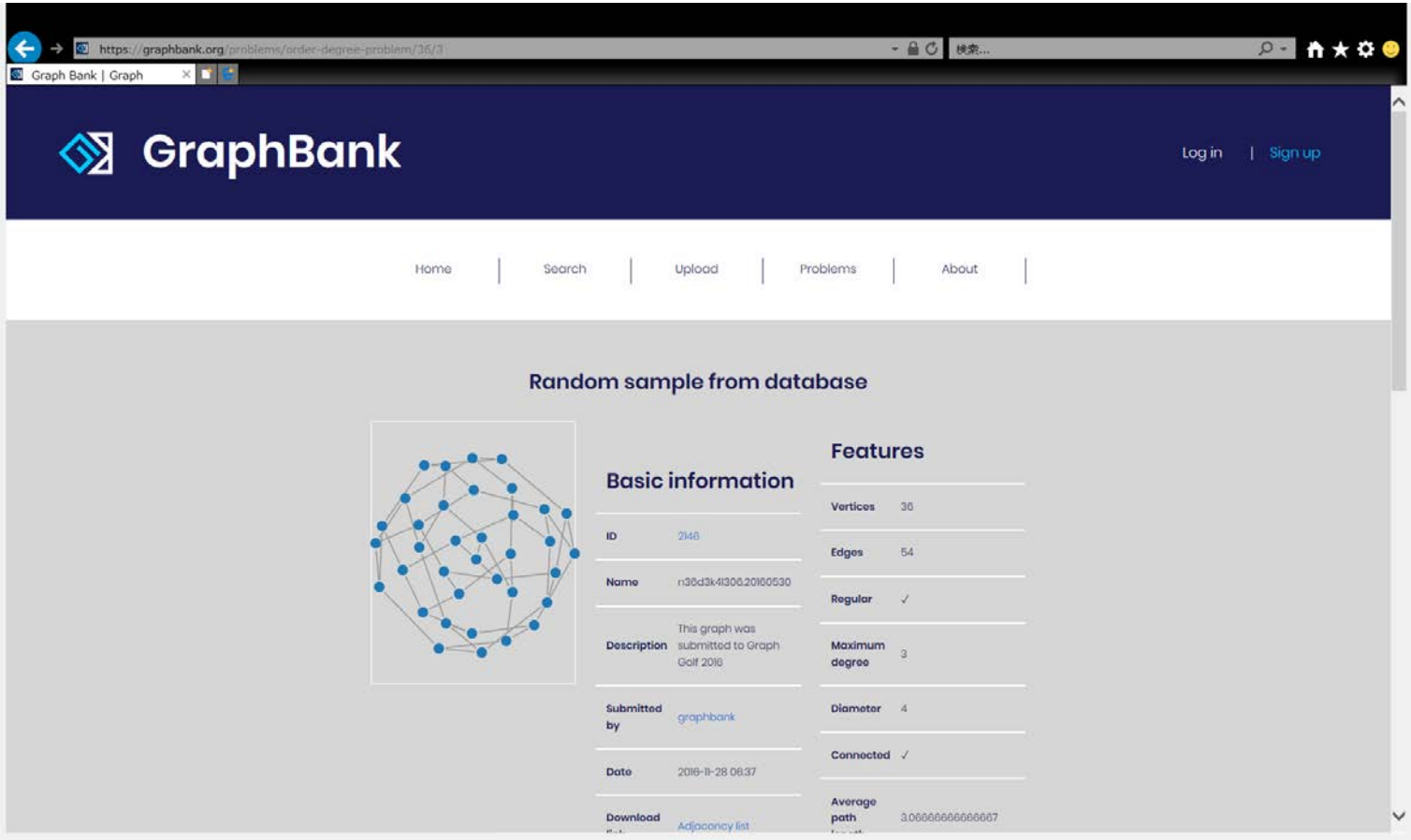
- Who achieve the smallest ASPL gap over all the order/degree pairs
- Effectively, who achieve the ASPL gap = 0

http://freedesignfile.com/9420-trophy-cup-and-medals-vector-set-05

Graph Bank

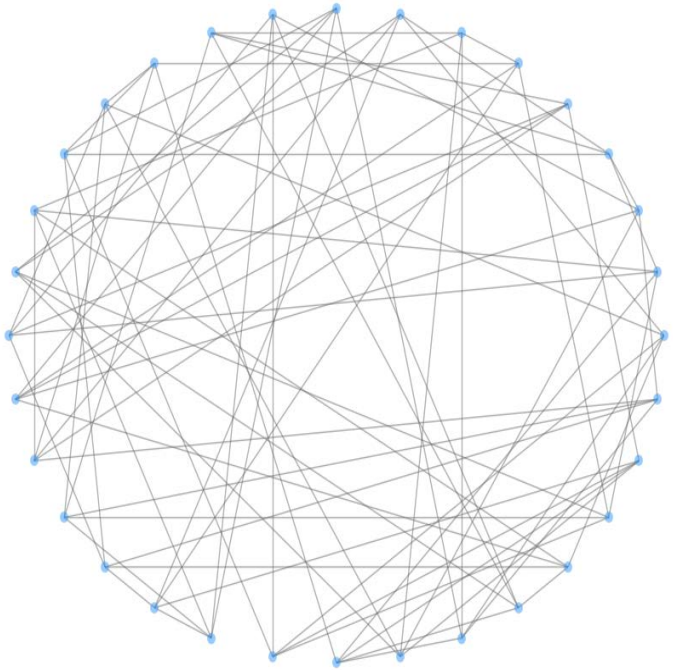
- Submitted graphs will be archived in Graph Bank, an online database of (interesting) graphs

https://graphbank.org



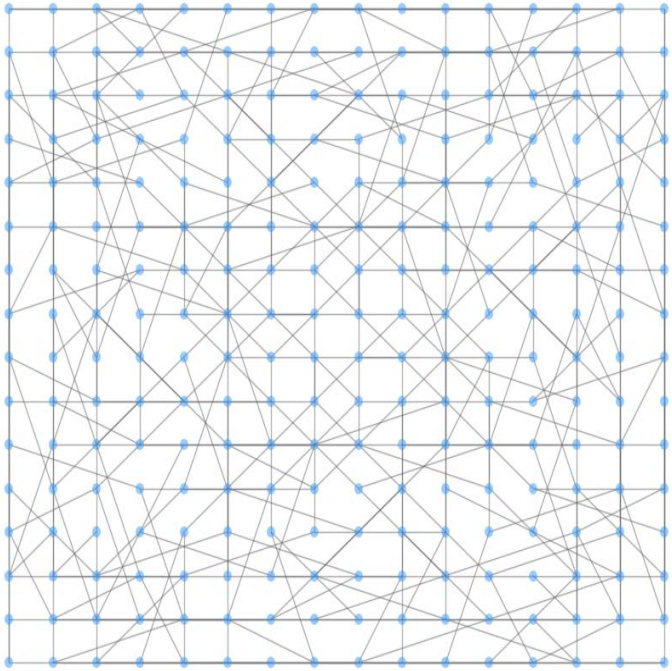
2017 New Categories

General Graph



- Order n
- Degree d

Grid Graph



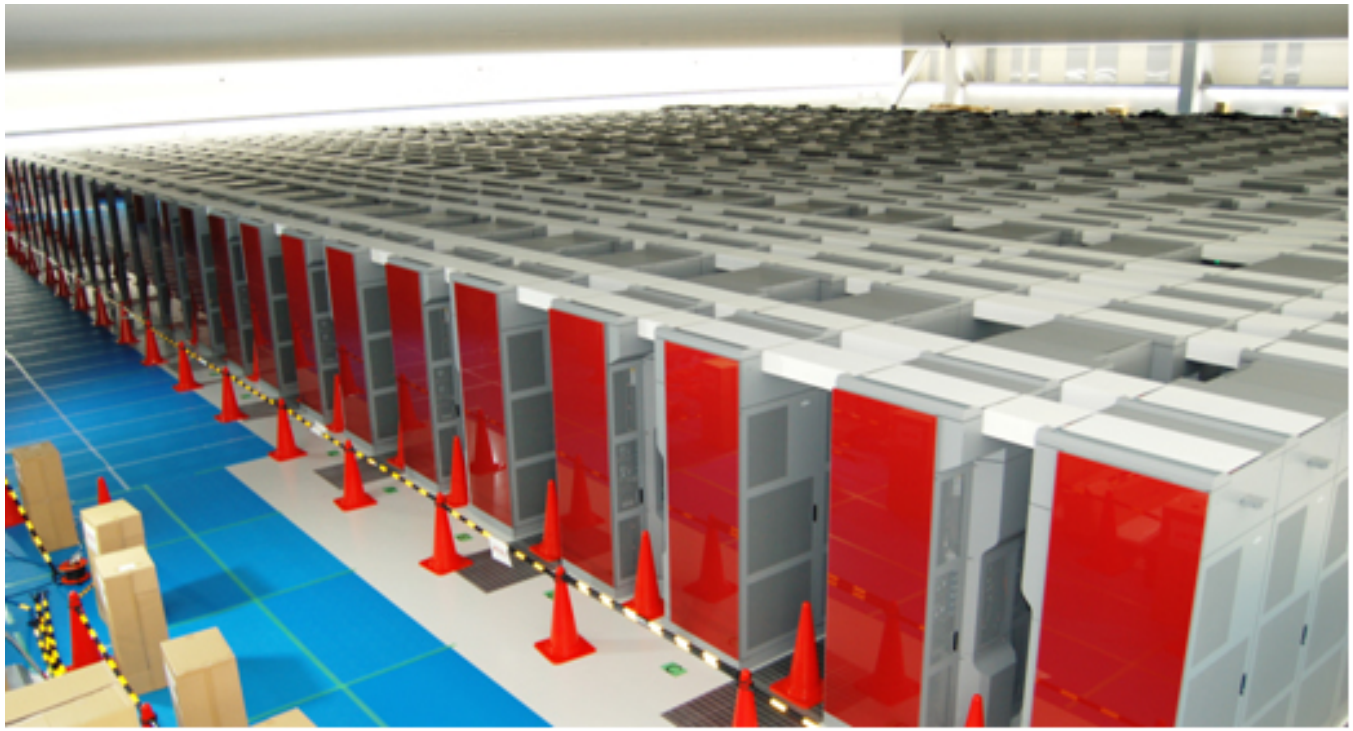
- Order n
- Degree d
- Max length r
  (in Manhattan distance)

Featured General Graph Parameters

Order	Degree	Corresponding Supercomputer	Topology	Top500 rank (Nov. 2016)
32	5			
256	18			
576	30	*Cori*, Cray XC40	Dragonfly	5
1344	30	*Piz Daint*, Cray XC50	Dragonfly	8
4896	24	*Tianhe-2*	Fat Tree	2
9344	10	*Titan*, Cray XK7	3-D Torus	3
88128	12	*K computer*	6-D Torus	7
98304	10	*Sequoia*, BlueGene/Q	5-D Torus	4
100000	32			
100000	64			

Featured Grid Graph Parameters

Grid	Order	Degree	Length
4×4	16	3	2
16×16	256	{3, 6, 15}	{3, 4, 15}
100×100	10000	{3, 9, 28}	{6, 18, 33}



© Riken, Fujitsu

2017 Final Results

Among 269 valid submissions from 20 unique participants

Best General Graphs

Order	Degree	Diam. (gap)	ASPL (gap)	Corresponding Supercomputer
32	5	**3 (0)**	**2.03226 (0)**	
256	18	3 (1)	1.93860 (0.00919)	
576	30	**2 (0)**	**1.94783 (0)**	*Cori*, Cray XC40
1344	30	**3 (0)**	2.34582 (0.03830)	*Piz Daint*, Cray XC50
4896	24	4 (1)	2.89823 (0.02080)	*Tianhe-2*
9344	10	**5 (0)**	4.24654 (0.23338)	*Titan*, Cray XK7
88128	12	6 (1)	4.88278 (0.10201)	*K computer*
98304	10	7 (1)	5.35521 (0.19984)	*Sequoia*, BlueGene/Q
100000	32	**4 (0)**	3.70556 (0.03388)	
100000	64	4 (1)	3.01537 (0.05697)	

Best Grid Graphs

Order	Degree	Length	Diam. (gap)	ASPL (gap)
16	3	2	**3 (0)**	**2.20000 (0)**
256	3	3	10 (3)	5.92016 (0.33192)
256	3	4	9 (2)	5.75300 (0.16477)
256	3	15	8 (1)	5.66725 (0.07901)
256	6	3	10 (6)	4.15570 (1.04982)
256	6	4	8 (4)	3.58100 (0.47512)
256	6	15	**4 (0)**	3.19069 (0.08480)
256	15	3	10 (7)	3.91336 (1.85453)
256	15	4	8 (5)	3.12197 (1.06314)
256	15	15	**3 (0)**	2.17929 (0.12047)
10000	3	6	33 (21)	14.84790 (4.07292)
10000	3	18	17 (5)	11.43242 (0.65744)
10000	3	33	16 (4)	11.24906 (0.47408)
10000	9	6	33 (28)	11.58996 (7.19122)
10000	9	18	11 (6)	5.15137 (0.75263)
10000	9	33	7 (2)	4.56047 (0.16173)
10000	28	6	33 (30)	11.53349 (8.61470)
10000	28	18	11 (8)	4.39407 (1.47528)
10000	28	33	6 (3)	3.25805 (0.33926)

General Graph Rankings

- Widest Improvement Ranking

	Author	#Best solutions
1	Takayuki Matsuzaki, Teruaki Kitasuka, Masahiro Iida	5
2	Ryuhei Mori	3
3	Ibuki Kawamata	2

- Deepest Improvement Ranking

	Author	ASPL gap
1	Ryuhei Mori	0
2	Takayuki Matsuzaki, Teruaki Kitasuka, Masahiro Iida	0.020800870569542163
3	Ibuki Kawamata	0.033880338803388366

Grid Graph Ranking

- Widest Improvement Ranking

	Author	#Best solutions
1	Ibuki Kawamata	8
2	Nakano	7
3	Takayuki Matsuzaki, Teruaki Kitasuka, Masahiro Iida	5

- Deepest Improvement Ranking

	Author	ASPL gap
1	Takayuki Matsuzaki, Teruaki Kitasuka, Masahiro Iida	0
1	Nakano	0
3	Ibuki Kawamata	0.16172709270927044

Award Ceremony

General Graph Widest and Grid Graph Deepest Improvement Awards

Takayuki Matsuzaki*
Teruaki Kitasuka†
Masahiro Iida*

*Kumamoto University
†Hiroshima University

General Graph Deepest Improvement Award

Ryuei Mori

Tokyo Institute of Technology

Grid Graph Widest Improvement Award

Ibuki Kawamata

Tohoku University

Discussion

Toward Graph Golf 2018

- Was the grid graph category a good challenge?
- How can we attract more overseas participants?
- Any questions, comments or suggestions?

Timetable

- 13:15–13:25 Intro & Award Ceremony
- 13:25–14:15 Keynote
- 14:15–14:30 Break
- 14:30–15:00 Talk 1
- 15:00–15:30 Talk 2
- 15:30–15:45 Discussion