
Workshop in CANDAR'18, Hida Takayama, Gifu, Japan 2018-11-27
Graph Golf 2018
The Order/degree Problem Competition

Timetable

- 13:30-13:50 Introduction & Award ceremony
- 13:50-14:20 Keynote *by Takeru Inoue*
- 14:20-14:40 Winner's talk *by Masahiro Nakao*
- 14:40-15:00 Winner's talk *by Toru Koizumi*
- 15:00-15:20 Contributor's talk *by Masato Haruishi*
- 15:20-15:30 Organizer's talk *by Michihiro Koibuchi*

Introduction

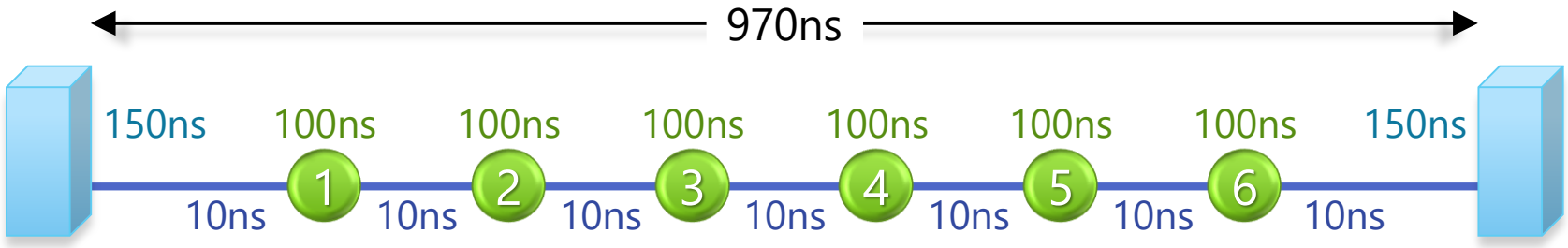
Graph Golf

- An online competition for the order/degree problem
- Aims at:
  - Building a catalog of "best" graphs
  - Sharing knowledge for creating small-diameter graphs
  - Connecting graph-theoreticians and computer engineers
- Past workshops
  - CANDAR '15, '16, '17
  - Intl. Symp. on Networks-on-Chip (NOCS 2016)
  - FIT 2016 (Japanese)
- NII News Release (Japanese)
  - https://www.nii.ac.jp/news/release/

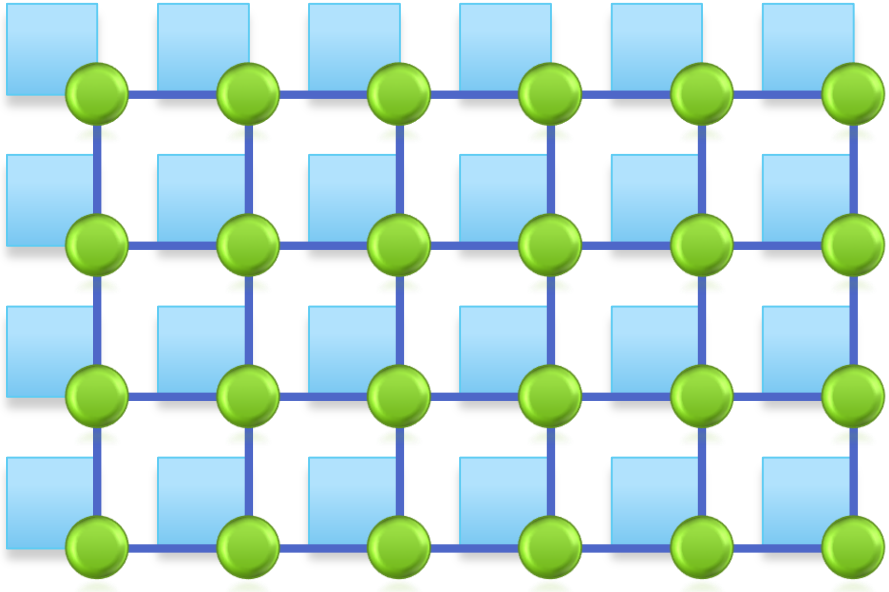
Winners of the 2015 competition

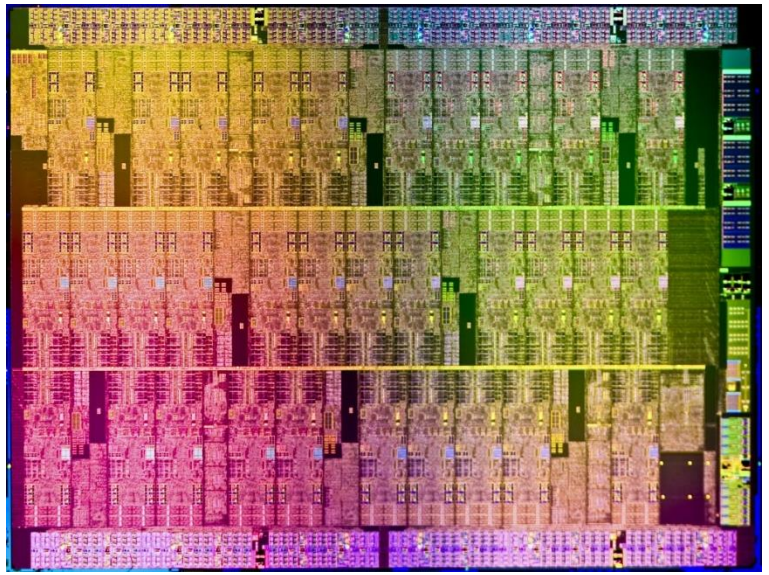
Background

- Low-latency networks are desired in supercomputers
  - 1μs for 3M cores ≒ **7 hops for 100k nodes**



- On-chip networks will soon face the same problem





https://www.extremetech.com/extreme/171678-intel-unveils-72-core-x86-knights-landing-cpu-for-exascale-supercomputing

Degree/Diameter Problem (DDP)

- Given a diameter k and a degree d, find a graph with the largest order n.
  - Known solutions are listed in the *Combinatorics Wiki*

http://combinatoricswiki.org/wiki/The_Degree_Diameter_Problem_for_General_Graphs

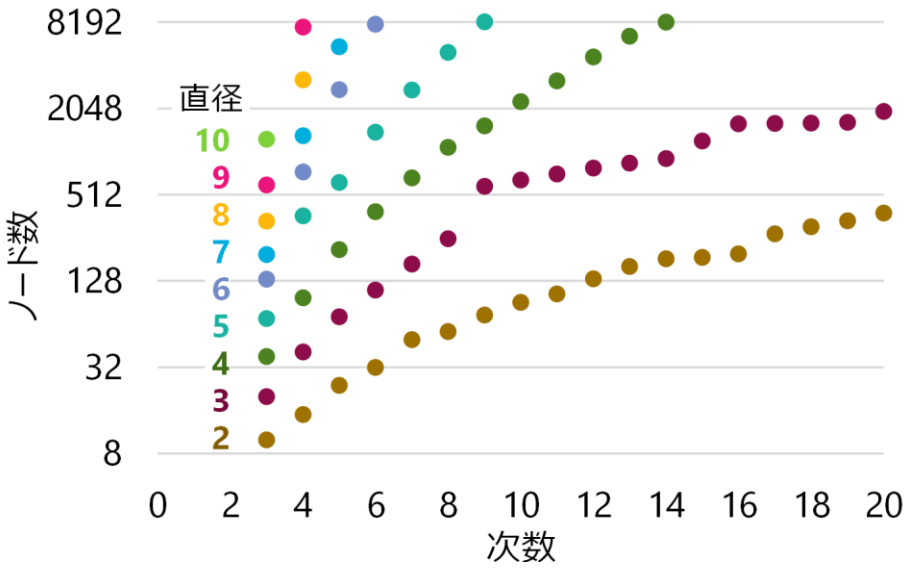


Table of the orders of the largest known graphs for the undirected degree diameter problem

$d \backslash k$	2	3	4	5	6	7	8	9	10
3	10	20	38	70	132	196	336	600	1 250
4	15	41	98	364	740	1 320	3 243	7 575	17 703
5	24	72	212	624	2 772	5 516	17 030	57 840	187 056
6	32	111	390	1 404	7 917	19 383	76 461	331 387	1 253 615
7	50	168	672	2 756	11 988	52 768	249 660	1 223 050	6 007 230
8	57	253	1 100	5 060	39 672	131 137	734 820	4 243 100	24 897 161
9	74	585	1 550	8 268	75 893	279 616	1 697 688	12 123 288	65 866 350
10	91	650	2 286	13 140	134 690	583 083	4 293 452	27 997 191	201 038 922
11	104	715	3 200	19 500	156 864	1 001 268	7 442 328	72 933 102	600 380 000
12	133	786	4 680	29 470	359 772	1 999 500	15 924 326	158 158 875	1 506 252 500
13	162	851	6 560	40 260	531 440	3 322 080	29 927 790	249 155 760	3 077 200 700
14	183	916	8 200	57 837	816 294	6 200 460	55 913 932	600 123 780	7 041 746 081
15	187	1 215	11 712	76 518	1 417 248	8 599 986	90 001 236	1 171 998 164	10 012 349 898
16	200	1 600	14 640	132 496	1 771 560	14 882 658	140 559 416	2 025 125 476	12 951 451 931
17	274	1 610	19 040	133 144	3 217 872	18 495 162	220 990 700	3 372 648 954	15 317 070 720
18	307	1 620	23 800	171 828	4 022 340	26 515 120	323 037 476	5 768 971 167	16 659 077 632
19	338	1 638	23 970	221 676	4 024 707	39 123 116	501 001 000	8 855 580 344	18 155 097 232
20	381	1 958	34 952	281 820	8 947 848	55 625 185	762 374 779	12 951 451 931	78 186 295 824

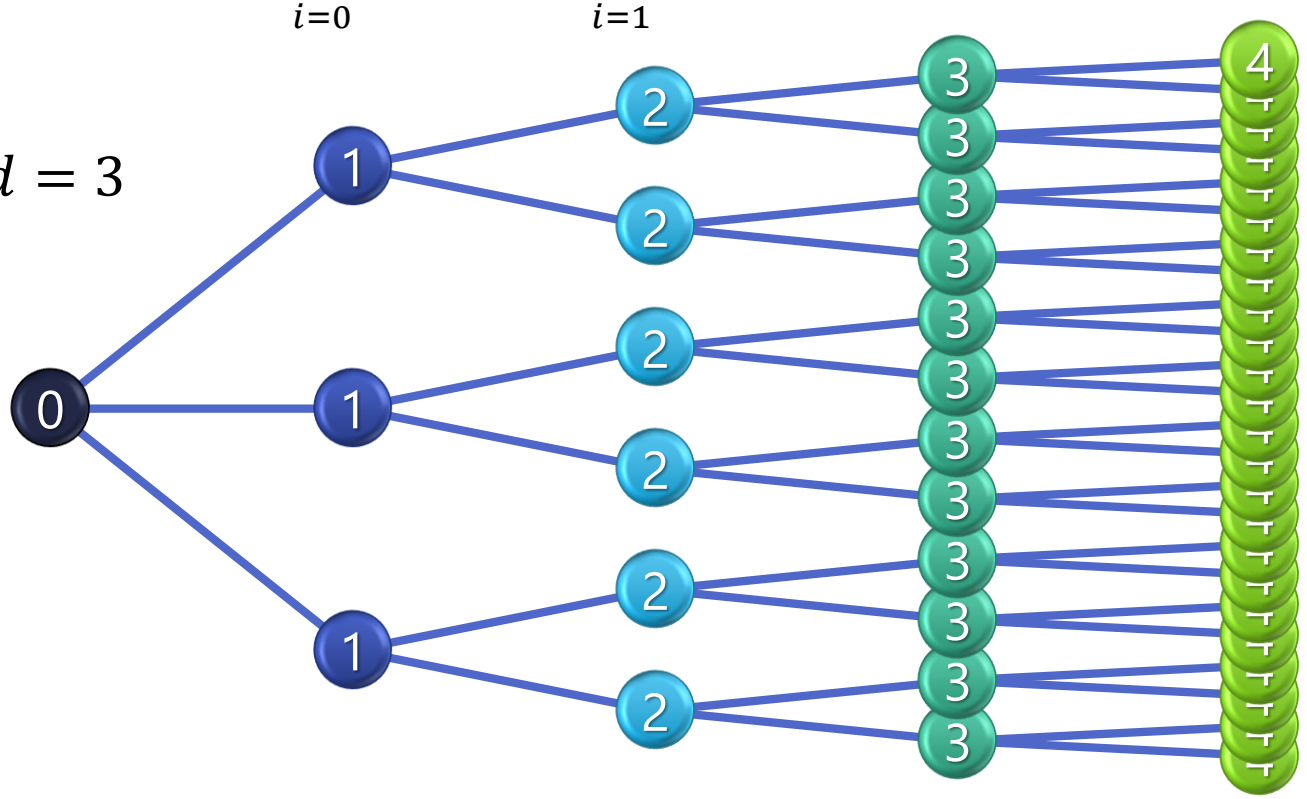
- Solutions given are not very useful for interconnects
  - Because these orders don't fit hardware design

Moore Bound for DDP

- Given diameter k and degree d, the upper bound of the order n is

$$N_{d,k} = \sum_{i=0}^{k} n_i = 1 + d\sum_{i=1}^{k} (d-1)^{i-1}$$

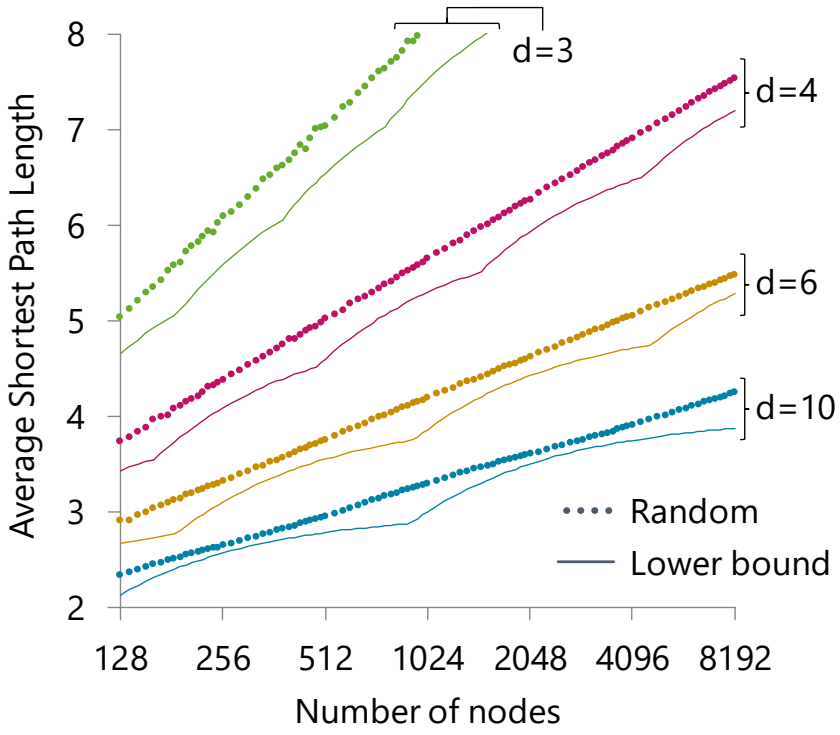
- Example for $d = 3$



Order n_k =	1	3	6	12	24
U.B. $N_{d,k}$ =	1	4	10	22	46

Order/Degree Problem (ODP)

- Given an order n and a degree d, find the graph with the smallest diameter k.
  - Among those with the same diameter, find the one with the smallest ASPL l.
- Gives useful solutions to design interconnects
  - However, theoreticians are not interested in ODP (why?)
- There can be better graphs than the random graphs



Moore Bound for ODP

- The lower bound $K_{n,d}$ of diameter k for order n and degree d

$$K_{n,d} = \begin{cases} \left\lceil \frac{n-1}{2} \right\rceil & \text{if } d = 2 \\ \left\lceil \log_{d-1}\left(\frac{(n-1)(d-2)}{d} + 1 \right) \right\rceil & \text{if } d > 2 \end{cases}$$

- The lower bound $L_{n,d}$ of ASPL l for order n and degree d

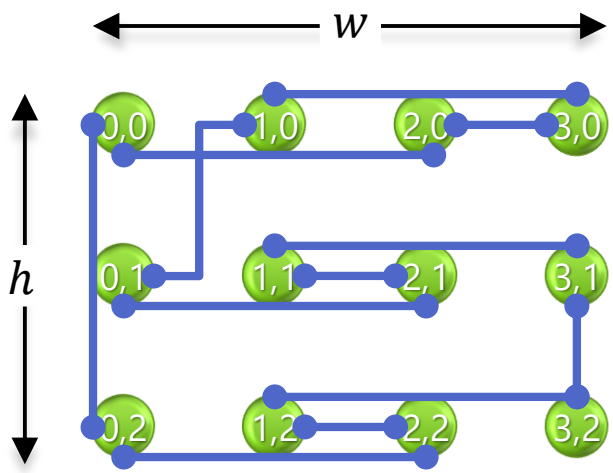
$$L_{n,d} = \begin{cases} 1 & \text{if } K_{n,d} = 1 \\ \frac{\sum_{i=1}^{K_{n,d}-1} id(d-1)^{i-1} + K_{n,d}\left(n - 1 - \sum_{i=1}^{K_{n,d}-1} d(d-1)^{i-1}\right)}{n-1} & \text{if } K_{n,d} \geq 2 \end{cases}$$

- ASPL gap p

$$p = \frac{l - L_{n,d}}{L_{n,d}}$$

ODP on Grid Graphs

- Given a degree d, find the graph with the smallest diameter k on a $w \times h$ grid, while keeping the lengths of each edge $\leq r$ in a Manhattan distance
  - Example: $w \times h = 4 \times 3,\ d = 2,\ r = 2$



Award Rules

Widest
Improvement Award

- Who find the largest number of "best" solutions
- The best solution means a graph with the smallest diameter, and with the smallest ASPL among those with the same diameter, for each order/degree pair.

Deepest
Improvement Award

- Who achieve the smallest ASPL gap over all the order/degree pairs
- Effectively, who achieve the ASPL gap = 0

http://freedesignfile.com/9420-trophy-cup-and-medals-vector-set-05

2018 Problems & Rankings

The 2018 Problems

- General Graph Category

n	d	
72	4	Real on-chip network
256	5	
256	10	
2300	10	DDP (2286, 10)
3019	30	Real supercomputer
4855	30	Real supercomputer
12000	7	DDP (11988, 7)
20000	11	DDP (19500, 11)
40000	8	DDP (39672, 8)
77000	6	DDP (76461, 6)
132000	8	DDP (131137, 8)
200000	32	
200000	64	
400000	32	

- Grid Graph Category

$w \times h$	d	r	
4×16	4	4	For practice
32×32	4	3	Length-bounded
32×32	4	4	Balanced
32×32	4	5	Moore-bounded
16×64	4	4	Length-bounded
16×64	4	5	Balanced
16×64	4	7	Moore-bounded
4×256	4	12	Length-bounded
4×256	4	18	Balanced
4×256	4	24	Moore-bounded

Widest Improvement Rankings

- General Graph

Rank	Authors	#Best solutions
1	Masahiro Nakao	8
2	Masato Haruishi	6
3	Toru Koizumi	1
3	Teruaki Kitasuka, Masahiro Iida	1

- Grid Graph

Rank	Authors	#Best solutions
1	EvbCFfp1XB	5
1	Nakano *	5

* Nakano is one of the organizers and excluded from the awards

Deepest Improvement Rankings

- General Graph

Rank	Authors	ASPL gap
1	Masahiro Nakao	0
1	Toru Koizumi	0
1	Teruaki Kitasuka, Masahiro Iida	0
4	Masato Haruishi	0.000481028525212 1

- Grid Graph

Rank	Authors	ASPL gap
1	EvbCFfp1XB	0.1125992063492065
2	Nakano *	0.6503390762463344

* Nakano is one of the organizers and excluded from the awards

Remarkable Achievements

- Featured Graphs

n	d	Diam. k	ASPL l	Diam. gap	ASPL gap	Found by
72	4	4	2.9859154929	0	0	Masahiro Nakao
256	5	5	3.4931372549		196	Masahiro Nakao
256	10	3	2.568627450			3 challengers
2300	10	5	3.5876468029		9643	Masahiro Nakao
3019	30	3	2.6932289447	0	0.0013800381	Masato Haruishi
4855	30	4	2.8088864673	1	0.0004810285	Masato Haruishi
12000	7	7	5.1		640220018	Masato Haruishi
20000	11	6	4.4		1226286314	Masahiro Nakao
40000	8	7	5.4650116252	1	0.1184279606	Masato Haruishi

Optimizes a real on-chip network!

Optimizes a real supercomputer!

- Non-featured Graphs

n	d	Diam. k	ASPL l	Diam. gap	ASPL gap	Found by
2394	10	4	3.6005014625	0	0.0267446719	Teruaki Kitasuka
20468	11	5	4.46		69849025	Teruaki Kitasuka
80050	6	8	6.86		83101600	Teruaki Kitasuka
137745	8	7	6.1304594029	1	0.3202026948	Teruaki Kitasuka

Beat the best-known DDP solutions!!!

Award Ceremony

General Graph Widest and Deepest Improvement Award

Masahiro Nakao

RIKEN Center for Computational Science

General Graph Deepest Improvement Award

Toru Koizumi

The University of Tokyo

General Graph Deepest Improvement Award

Teruaki Kitasuka
Hiroshima University
and
Masahiro Iida
Kumamoto University

Grid Graph Widest and Deepest Improvement Award

EvbCFfp1XB

Congratulations!!

Graph Golf 2018
Organizing Committee