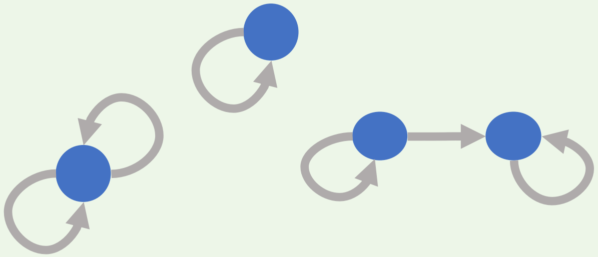
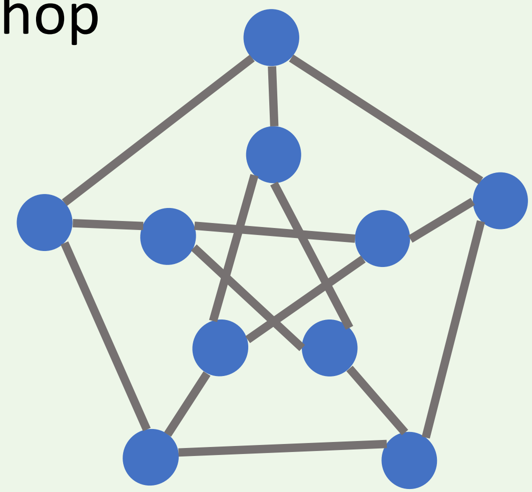


Search voltage graph for order degree problem

Masato Haruishi

CANDAR2018, Graph Golf workshop

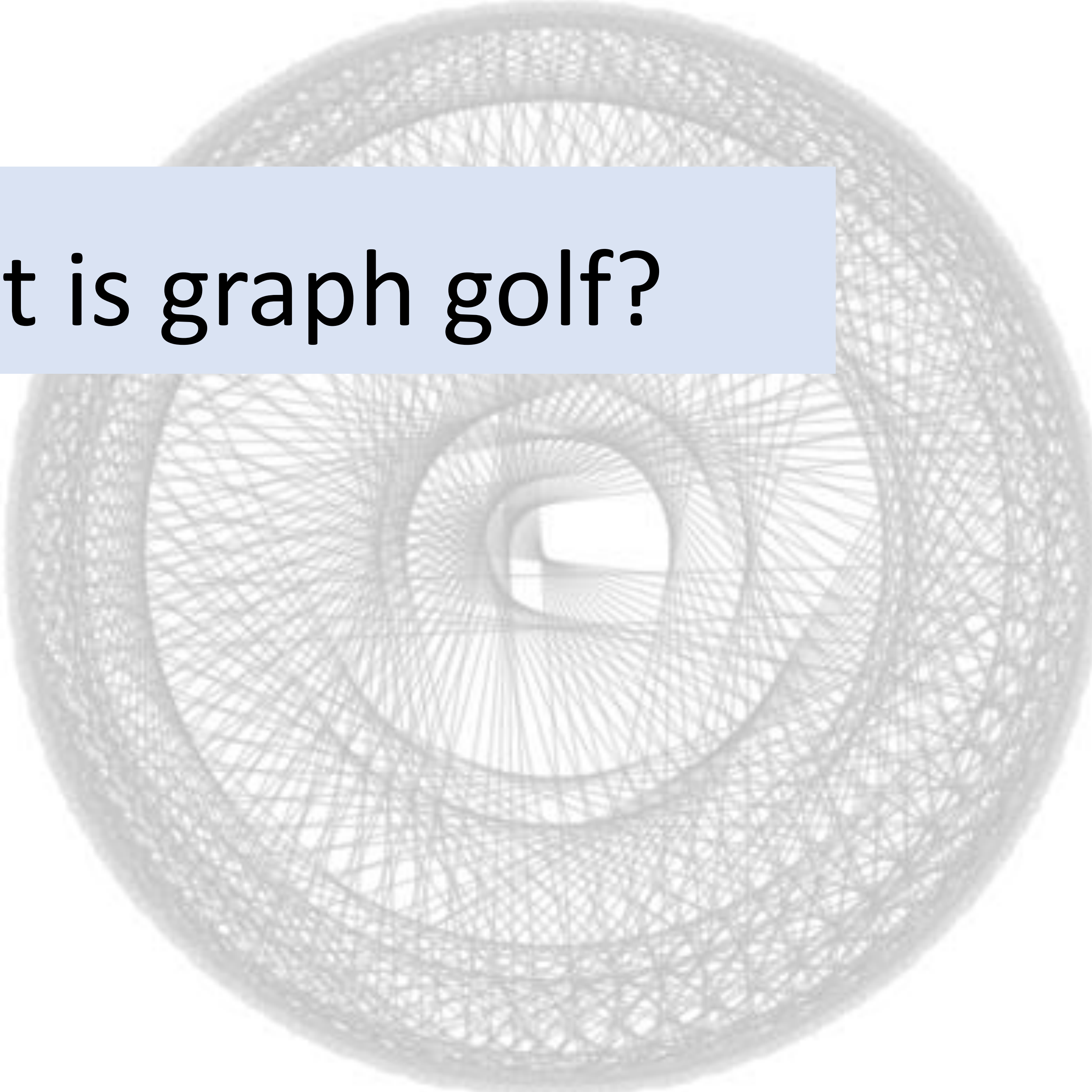
27 Nov. 2018



Outline

1. What is graph golf?
2. Our graphs
3. Voltage graph
4. Our strategy
    1. Strengths
    2. Overview
    3. Flowchart
5. Source code

1.What is graph golf?



What is graph golf?

Graph Golf is an order/degree problem.

- The order/degree problem with parameters n and d: Find a graph with minimum diameter over all undirected graphs with the number of vertices = n and $degree \leq d$.
- The person who looks for smaller **ASPL** and **diameter** will win.
- There are general graph and grid graph categories.
- I joined the general category.

Graph Golf

Glossary

ASPL (average shortest path length):

The average of the shortest path lengths of all vertex combinations.

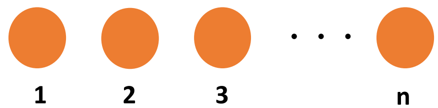
Diameter:

Maximum vertex distance of graph.

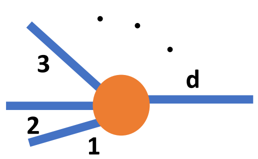
Definition of graph

Graph : $G = (V, E)$

Order : n



Degree : d



Undirected and unweighted

Shortest path length : $\mathrm{s}(v_1, v_2)$ for $v_1, v_2 \in V$

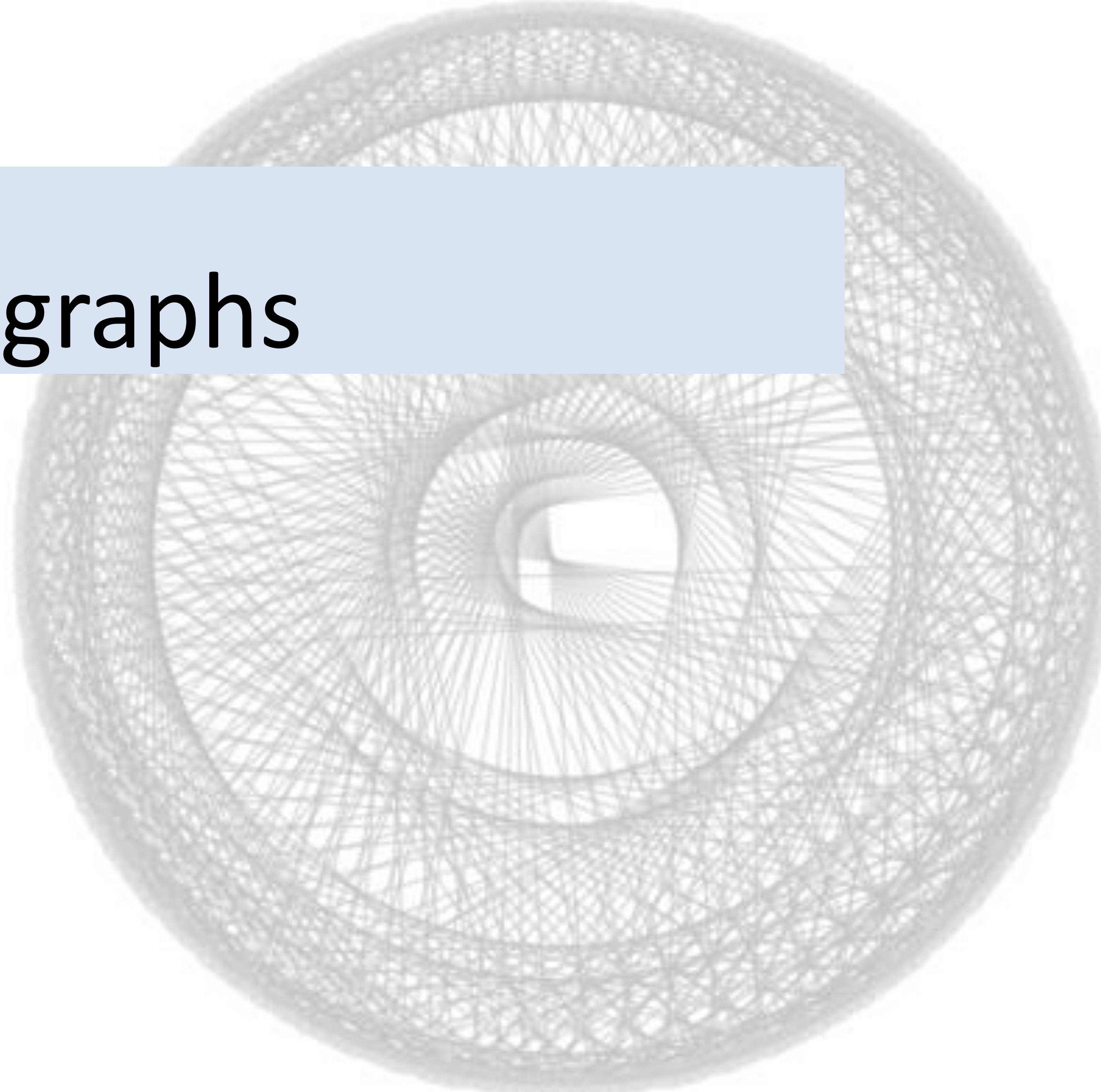
Diameter : k = max{s(v_1, v_2)| $v_1, v_2 \in V$}

Average shortest path length :

L = average{s(v_1, v_2)| $v_1, v_2 \in V, v_1 \neq v_2$}

Graph Golf

2.Our graphs



Results

General Graph Widest Improvement ranking

	Rank	Author	Number of best solutions
🏆	1	Masahiro Nakao	8
	2	haruishi masato	6
	3	Toru Koizumi	1
	3	Teruaki Kitasuka, Masahiro Iida	1

General Graph Deepest Improvement ranking

	Rank	Author	ASPL gap
🏆	1	Masahiro Nakao	0.0
🏆	1	Toru Koizumi	0.0
🏆	1	Teruaki Kitasuka, Masahiro Iida	0.0
	4	haruishi masato	0.000481028525212146

I won the second place of general graph widest improvement ranking and fourth place of general graph deepest improvement ranking.

Graph Golf

Results

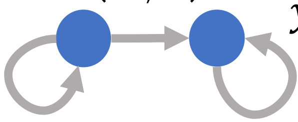
Order n	Degree d	Diam. k	ASPL l	ASPL gap
72	4	4	2.98592	0.00000
256	5	5	3.49314	0.02255
256	10	3	2.56863	0.00000
2300	10	5	3.58765	0.03132
3019	30	3	2.69323	0.00138
4855	30	4	2.80889	0.00048
12000	7	7	5.17601	0.26402
20000	11	6	4.44389	0.12263
40000	8	7	5.46501	0.11843
77000	6	9	6.83465	0.21499
132000	8	8	6.09465	0.29266
200000	32	5	3.84909	0.01326
200000	64	4	3.23627	0.25707
400000	32	5	3.99682	0.07890

I submit six best graphs such as a graph of order 3019 and degree 30.

Graph Golf

Results

(17,23)
(35,15)
(35,20)

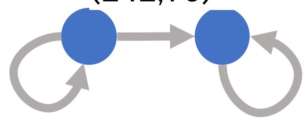


(22,22)
(12,37)

(6,68)
(11,26)

$n = 12000,$
$d = 7,$
$x = 75,$
$y = 80,$
$z = 17$

(153,42)
(155,45)
(160,44)
(242,73)



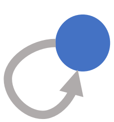
(1,35)
(9,73)

(137,63)
(100,9)

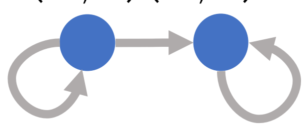
$n = 40000,$
$d = 8,$
$x = 125,$
$y = 160,$
$z = 18$

(16,71) (3,41)
(9,53) (30,4)
(0,72) (28,69)
(23,2) (6,24)
(7,49) (1,70)
(17,36) (27,31)
(36,78) (9,52)
(30,16) (36,10)
(34,16) (33,68)
(32,36) (27,48)
(24,24) (21,22)
(20,11) (17,9)
(13,4) (9,79)
(8,51) (8,49)
(5,67) (0,38)

$n = 3020,$
$d = 30,$
$x = 151,$
$y = 20,$
$z = 19$



(14,45) (3,69)
(10,76) (2,35)
(25,75) (27,15)
(18,70) (22,72)
(29,29) (4,68)
(13,71) (9,69)
(20,58) (23,33)

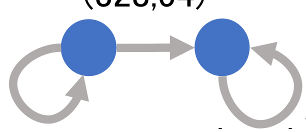


(26,9)
(18,63)
(11,8)
(13,45)
(29,41)
(0,16)
(8,59)
(2,78)

(16,59)
(7,20)
(27,47)
(7,24)
(3,43)
(2,41)
(26,1)
(9,39)

$n = 4860,$
$d = 30,$
$x = 135,$
$y = 18,$
$z = 4$

(277,60)
(328,54)



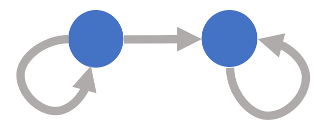
(477,59)
(458,8)

(81,78)
(446,3)

$n = 77000,$
$d = 6,$
$x = 275,$
$y = 140,$
$z = 73$

(234,10) (368,71)
(339,67) (1050,11)
(1029,18) (1160,39)
(175,70) (598,33)
(13,2) (469,6)
(135,54) (427,75)
(214,41) (117,3)
(566,79) (726,14)

$n = 200000,$
$d = 32,$
$x = 500,$
$y = 200,$
$z = 153$



(1,75)
(211,7)
(469,42)
(38,45)
(206,60)
(814,66)
(403,47)
(193,11)

(643,52)
(170,16)
(23,74)
(552,18)
(1006,37)
(197,36)
(1184,26)
(694,42)

Graph Golf

3.Voltage graph

Voltage graph

- a directed graph.
- contains edges labeled with voltage.

<u>Voltage graph examples</u>

Voltage graph : G'

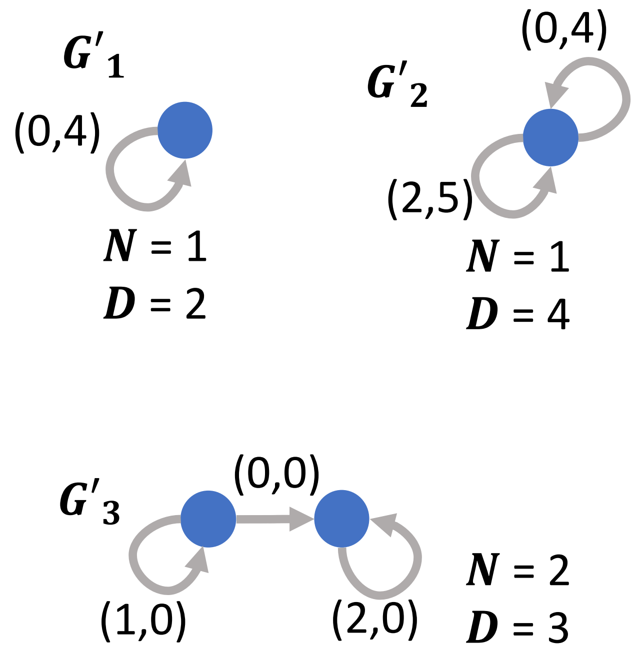
Vertices of voltage graph : V'

Edges of voltage graph : E'

The number of edge : N

Degree : $D = D_{in} + D_{out}$

Voltages are assigned



Graph Golf

How to make a derived graph

Order : $\boldsymbol{n}$ $\boxed{n = Nk \ (k \in \mathbb{N})}$

Degree : $\boldsymbol{d}$ $\boxed{d = D}$

Parameters : $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \mathbb{N}$ *such that*

$$\boxed{k = xy}$$

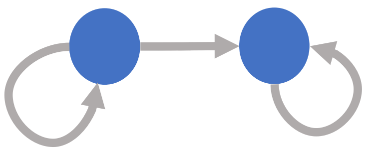
$$\boxed{z^y \equiv 1 \bmod x}$$

Voltages : $\boldsymbol{B}$ $\boxed{B = Z_x \otimes Z_y}$ $(Z_x = \{i|0 \le i \le x-1\},$ $Z_y = \{i|0 \le i \le y-1\})$

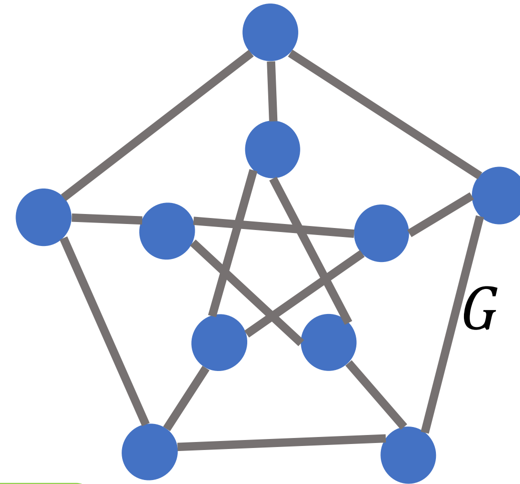
Assigned voltages : $\boldsymbol{A}$ $\boxed{\boldsymbol{A} \subseteq \boldsymbol{B}}$

Graph Golf

How to make a derived graph



$G' = (V', E')$



$G = (V, E)$

$$\boxed{\boldsymbol{V} = \boldsymbol{V}' \otimes \boldsymbol{B}} \tag{1}$$

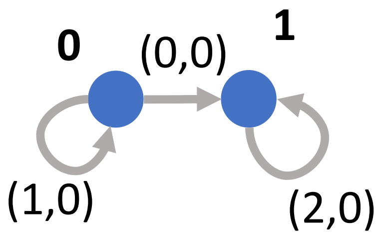
$$\boxed{\boldsymbol{E} = \left\{ (\boldsymbol{p}, \boldsymbol{g}), (\boldsymbol{q}, \boldsymbol{h}) \,\middle|\, \begin{array}{l} (\boldsymbol{p}, \boldsymbol{q}) \in \boldsymbol{E}'; \boldsymbol{g}, \boldsymbol{h} \in \boldsymbol{B}; \\ \boldsymbol{h} = \boldsymbol{g} \rtimes_{\boldsymbol{z}} \boldsymbol{a}; \boldsymbol{a} \in \boldsymbol{A} \end{array} \right\}} \tag{2}$$

The operator $\rtimes$ represents semi-direct product

Defined as $(a, b) \rtimes_e (c, d) = (a + e^b c, b + d)$

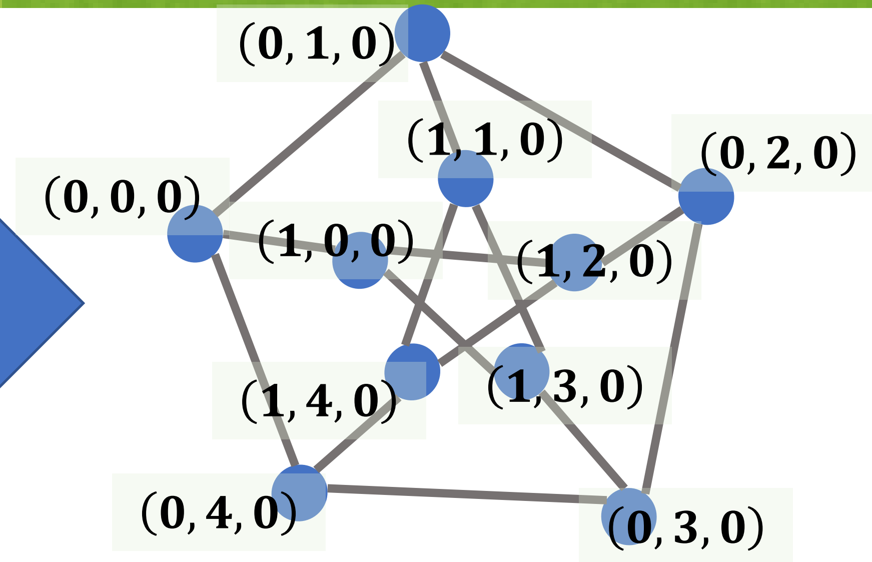
Graph Golf

How to make a derived graph



$$G' = (V', E')$$

Lift
x=5,y=1,z=1



$$G = (V, E)$$

$$B = \{(0,0), (1,0), (2,0), (3,0), (4,0)\}$$

$$A_{0\to 0} = \{(0,0)\}, A_{0\to 1} = \{(1,0)\}, A_{1\to 1} = \{(2,0)\}$$

$$V' = \{0,1\}$$

$$E' = \{0 \to 0; 0 \to 1; 1 \to 1\}$$

(1),(2),calculate

$$\boldsymbol{V} = \boldsymbol{V}' \otimes \boldsymbol{B} = \left\{ \begin{array}{l} (0,0,0), (0,1,0), (0,2,0), (0,3,0), (0,4,0), \\ (1,0,0), (1,1,0), (1,2,0), (1,3,0), (1,4,0) \end{array} \right\}$$

$$\boldsymbol{E} = \left\{ \begin{array}{l} (0,0,0) - (0,1,0); (0,0,0) - (1,0,0); \cdots \\ \cdots; (1,3,0) - (1,0,0); (1,4,0) - (1,1,0) \end{array} \right\}$$

How to calculate ASPL

- **Breadth-first search**

  you can calculate the distance to a vertex and all other vertices.

Pseudocode

```
function breadth-first search (v)
j = 0
V ← 1
    Add v to the queue
    While Queue is not empty do
        v ← Retrieve from Q
        for each Vertex i connected to v do
            if i not visited then
                Mark i as visited
                j = j + 1
                Add i to
```

Graph Golf

Time taken to calculate ASPL

In case of the random graph.

- In order to obtain ASPL, we perform a Breadth-first search at all nodes.
- The Breadth-first search consumes $\boldsymbol{O(nd)}$ time.
- Since it must be done on all nodes, ASPL consumes $\boldsymbol{O\left(n^2 d\right)}$ time.

Graph Golf

Time taken to calculate ASPL

In case of the derived graph.

- The derived graph are isomorphic from any vertex corresponding to the voltage graph.
- The Breadth-first search for obtaining the ASPL is only the number of vertices of the voltage graph. So, $\boldsymbol{O(nd)}$ time.

Graph Golf

4. Our strategy

Strengths

There are many sets of parameters.

For example, $\mathrm{N} = 2\ and\ n = 20000$, there are 7140 kinds, such as $(\mathrm{x}, y, z) = (200, 50, 31)$.

However, most sets can't make good graphs.

Therefore, calculate every parameters lightly.

Then choose a parameter that can make good graphs.

Search the selected parameters thoroughly.

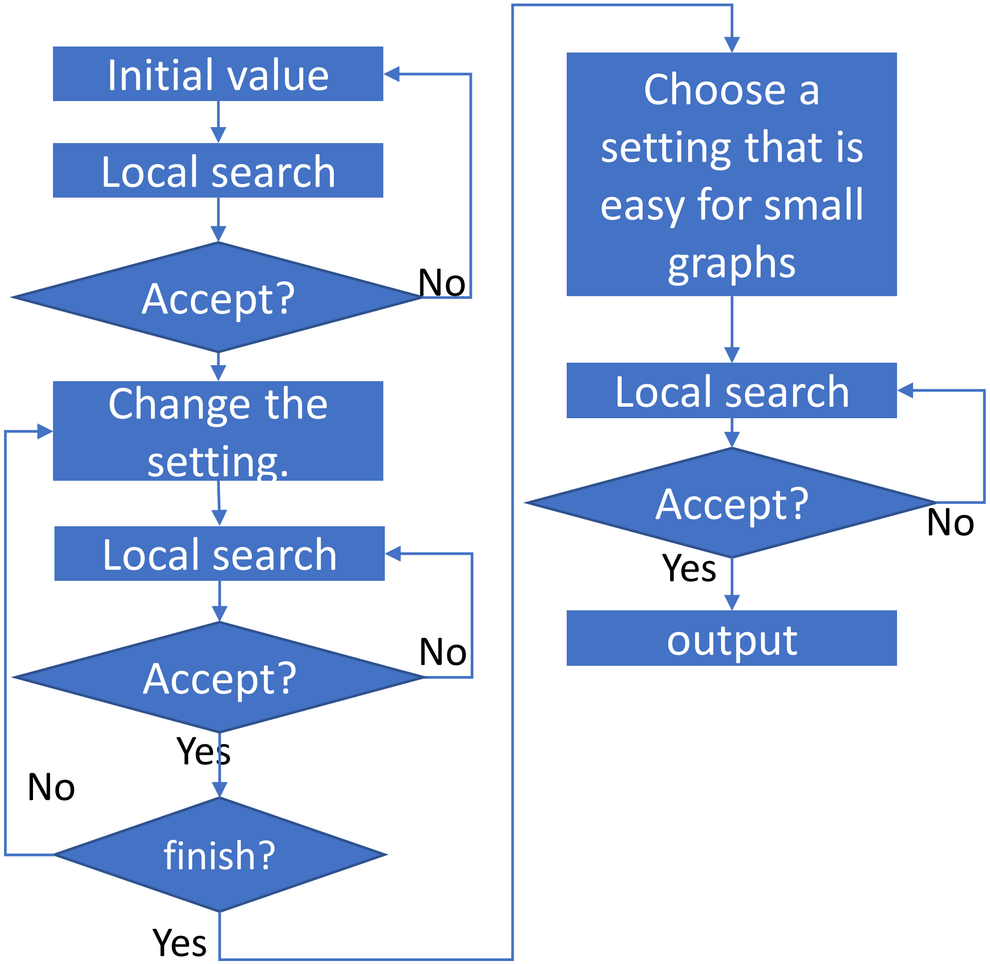
As a result, good graphs can be made with little time.

Overview

1. Determine the initial values.(parameters and voltage graph)
2. Randomly assign voltages to voltage graph.
3. Perform local search until convergence.
4. Do 2 and 3 several times.
5. Try all combinations of initial values.
6. Choose an initial values that can create a small graph.
7. Repeat steps 2 and 3 with the initial values found in 7.

Graph Golf

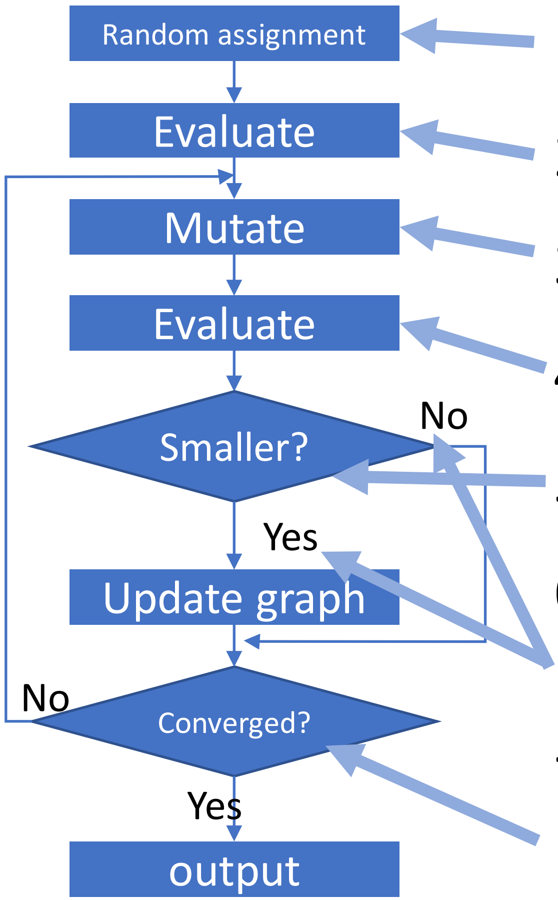
Flowchart



- This is the flowchart of my strategy.
- Search all parameters x, y and z to reduce uncertainty factors.

Graph Golf

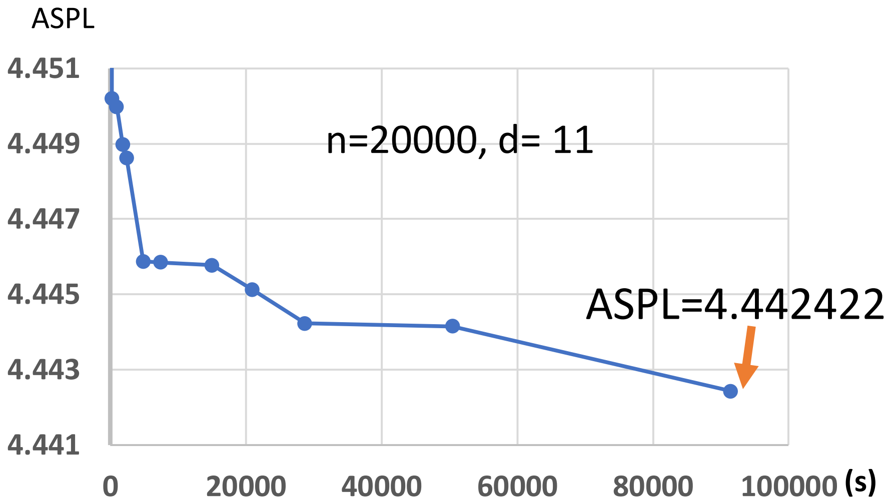
Local search

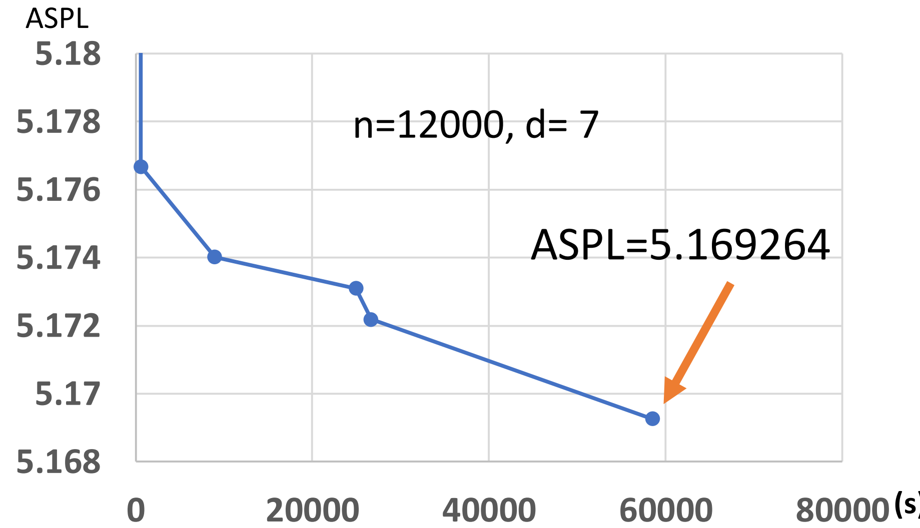


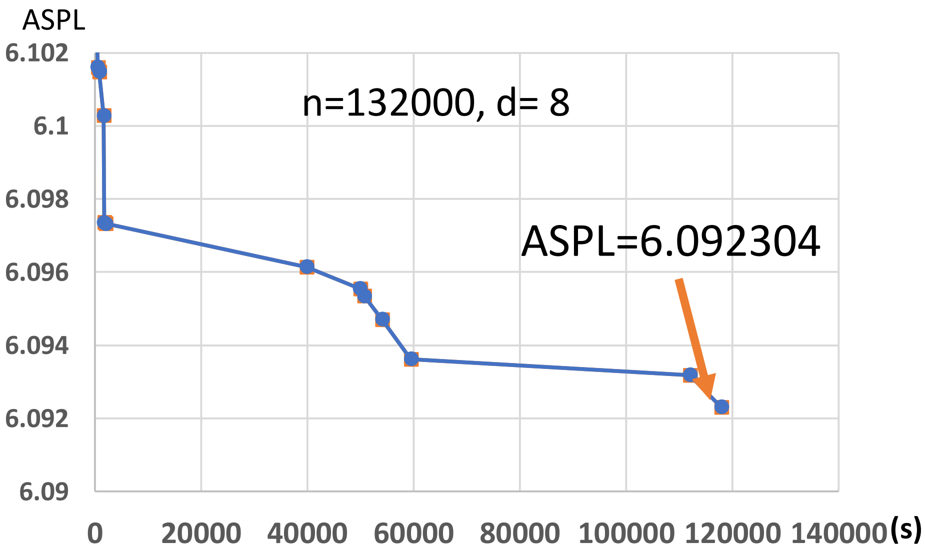
1. Generate a derived graph randomly.
2. Calculate the diameter of the graph and ASPL.
3. Make a little change in voltage set.
4. Calculate the diameter and ASPL.
5. Judged whether the graph has become small.
6. If it is smaller, update the graph. If it is big, cancel the change.
7. It is determined whether or not convergence has occurred.
8. If it is not the end go back to 3.

Graph Golf

Convergence







12000	7	7	5.17601	0.26402
20000	11	6	4.44389	0.12263
132000	8	8	6.09465	0.29266

Graph Golf

What I noticed (1/3)

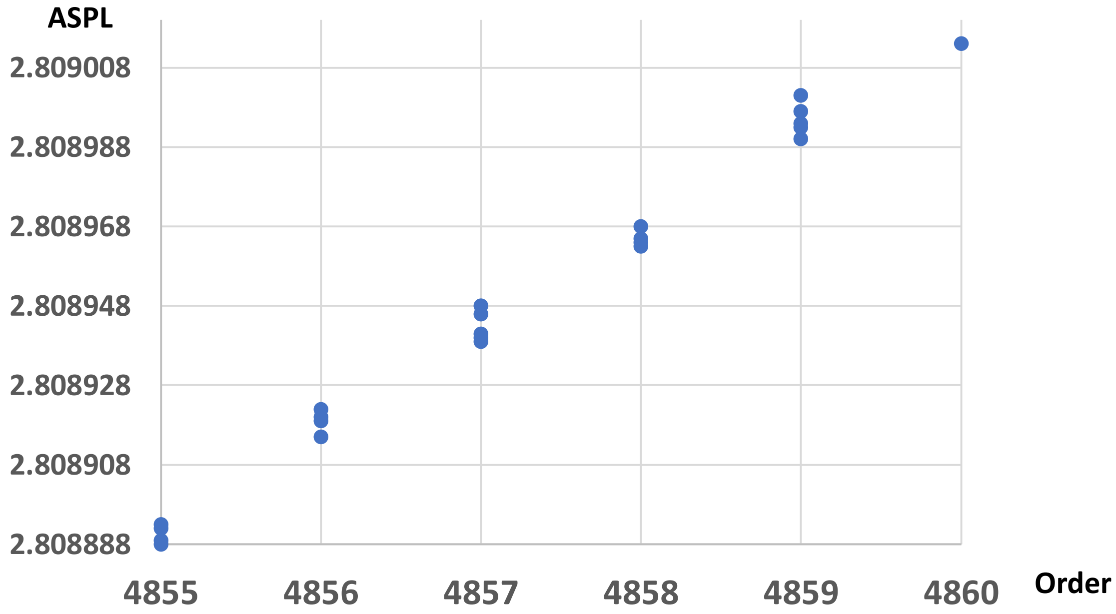
- In voltage graph, Orders with many divisors give good results.
- I could not make a good graph at n= 4855.
- However, n= 4860 got a good graph.

As the number of divisors increases, the number of parameter sets also increases.

Therefore, we predict that good graphs are easy to calculate.

What I noticed(2/3)

- Deleting one order lowers ASPL.



- At the time of z = 1, I could not get a good graph.

What I noticed (3/3)

【Which voltage graph is better?】

- The voltage graph I used is N = 2 and N = 1.
- In some orders, N = 4 or N = 8 was better.
  (Specifically, 256 is N = 4. 20000 is N = 8.)
- There were other good voltage graphs, but it was not enough time to find out properly.

5. Source code

Source code

【URL】

https://github.com/Haruishimasato/voltage-graph/tree/Haruishimasato-programs

- I release the program on GitHub.

References

[1] Teruaki Kitasuka, Takayuki Matsuzaki, and Masahiro Iida (2018), Order Adjustment Approach using Cayley Graphs for the Order/Degree Problem. advance publication, IEICE trans. on Information and Systems.DOI: 10.1587/transinf.2018PAP0008

[2]Ibuki Kawamata (2017), Approximate evaluation and voltage assignment for order/degree problem

Graph Golf workshop, CANDAR 2017. http://research.nii.ac.jp/graphgolf/2017/candar17/graphgolf2017-kawamata.pdf

Graph Golf