Search voltage graph for order degree problem

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Outline

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2. Our graphs
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4. Our strategy
   1. Strengths
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1. What is graph golf?
What is graph golf?

**Graph Golf** is an order/degree problem.

- The order/degree problem with parameters $n$ and $d$: Find a graph with minimum diameter over all undirected graphs with the number of vertices = $n$ and degree $\leq d$.
- The person who looks for smaller **ASPL** and **diameter** will win.
- There are general graph and grid graph categories.
- I joined the general category.
ASPL (average shortest path length):
The average of the shortest path lengths of all vertex combinations.

Diameter:
Maximum vertex distance of graph.
Definition of graph

Graph: \( G = (V, E) \)

Order: \( n \)

Degree: \( d \)

Undirected and unweighted

Shortest path length: \( s(v_1, v_2) \) for \( v_1, v_2 \in V \)

Diameter: \( k = \max\{s(v_1, v_2) \mid v_1, v_2 \in V\} \)

Average shortest path length:
\( L = \text{average}\{s(v_1, v_2) \mid v_1, v_2 \in V, v_1 \neq v_2\} \)
2. Our graphs
I won the second place of general graph widest improvement ranking and fourth place of general graph deepest improvement ranking.

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<tr>
<th>Rank</th>
<th>Author</th>
<th>Number of best solutions</th>
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<tr>
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<table>
<thead>
<tr>
<th>Rank</th>
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<th>ASPL gap</th>
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Results

I submit six best graphs such as a graph of order 3019 and degree 30.

<table>
<thead>
<tr>
<th>Order ( n )</th>
<th>Degree ( d )</th>
<th>Diam. ( k )</th>
<th>ASPL ( l )</th>
<th>ASPL gap</th>
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</table>
Results

\( n = 12000, \quad d = 7, \quad x = 151, \quad y = 20, \quad z = 17 \)

\( n = 3020, \quad d = 30, \quad x = 151, \quad y = 20, \quad z = 19 \)

\( n = 4860, \quad d = 30, \quad x = 135, \quad y = 18, \quad z = 4 \)

\( n = 77000, \quad d = 6, \quad x = 275, \quad y = 140, \quad z = 73 \)

\( n = 40000, \quad d = 8, \quad x = 125, \quad y = 160, \quad z = 18 \)

\( n = 200000, \quad d = 32, \quad x = 500, \quad y = 200, \quad z = 153 \)
3. Voltage graph
Voltage graph

- a directed graph.
- contains edges labeled with voltage.

Voltage graph examples

**Voltage graph :** $G'$

**Vertices of voltage graph :** $V'$

**Edges of voltage graph :** $E'$

**The number of edge :** $N$

**Degree :** $D = D_{in} + D_{out}$

Voltages are assigned

$G'_1$
- $(0,4)$
- $N = 1$
- $D = 2$

$G'_2$
- $(0,4)$
- $(2,5)$
- $N = 1$
- $D = 4$

$G'_3$
- $(0,0)$
- $(1,0)$
- $(2,0)$
- $N = 2$
- $D = 3$
How to make a derived graph

Order: \( n \)
\[ n = Nk \ (k \in \mathbb{N}) \]

Degree: \( d \)
\[ d = D \]

Parameters: \( x, y, z \in \mathbb{N} \) such that
\[ k = xy \]
\[ z^y \equiv 1 \mod x \]

Voltages: \( B \)
\[ B = Z_x \otimes Z_y \]
\( (Z_x = \{i|0 \leq i \leq x - 1\}, \ Z_y = \{i|0 \leq i \leq y - 1\}) \)

Assigned voltages: \( A \)
\[ A \subseteq B \]
How to make a derived graph

\[ G' = (V', E') \]

\[ V = V' \bigotimes B \]  \hspace{1cm} (1)

\[ E = \left\{ (p, g), (q, h) \mid (p, q) \in E'; g, h \in B; h = g \rtimes_z a; a \in A \right\} \] \hspace{1cm} (2)

The operator \( \rtimes \) represents semi-direct product

Defined as \( (a, b) \rtimes_e (c, d) = (a + e^b c, b + d) \)
How to make a derived graph

Let \( x = 5, y = 1, z = 1 \)

\[
G' = (V', E')
\]

\[
\begin{align*}
G &= (V, E) \\
V &= V' \boxtimes B = \\
&= \{(0,0,0), (0,1,0), (0,2,0), (0,3,0), (0,4,0), \\
&(1,0,0), (1,1,0), (1,2,0), (1,3,0), (1,4,0)\} \\
E &= \{(0,0,0) - (0,1,0); (0,0,0) - (1,0,0); \ldots; (1,3,0) - (1,0,0); (1,4,0) - (1,1,0)\}
\end{align*}
\]

\[
B = \{(0,0), (1,0), (2,0), (3,0), (4,0)\}
\]

\[
A_{0\to 0} = \{(0,0)\}, A_{0\to 1} = \{(1,0)\}, A_{1\to 1} = \{(2,0)\}
\]

\[
V' = \{0, 1\} \\
E' = \{0 \to 0; 0 \to 1; 1 \to 1\}
\]
How to calculate ASPL

• Breadth-first search
  you can calculate the distance to a vertex and all other vertices.

Pseudocode

1. function breadth-first search (v)
2. j = 0
3. V ← 1
4. Add v to the queue
5. While Queue is not empty do
6.   v ← Retrieve from Q
7.   for each Vertex i connected to v do
8.     if i not visited then
9.       Mark i as visited
10.    j = j + 1
11. Add i to
In case of the random graph.

- In order to obtain ASPL, we perform a Breadth-first search at all nodes.
- The Breadth-first search consumes $O(nd)$ time.
- Since it must be done on all nodes, ASPL consumes $O(n^2d)$ time.
In case of the derived graph.

• The derived graph are isomorphic from any vertex corresponding to the voltage graph.

• The Breadth-first search for obtaining the ASPL is only the number of vertices of the voltage graph. So, $O(nd)$ time.
4. Our strategy
There are many sets of parameters. For example, $N = 2$ and $n = 20000$, there are 7140 kinds, such as $(x, y, z) = (200, 50, 31)$. However, most sets can’t make good graphs. Therefore, calculate every parameters lightly. Then choose a parameter that can make good graphs. Search the selected parameters thoroughly. As a result, good graphs can be made with little time.
Overview

1. Determine the initial values (parameters and voltage graph).
2. Randomly assign voltages to voltage graph.
3. Perform local search until convergence.
4. Do 2 and 3 several times.
5. Try all combinations of initial values.
6. Choose an initial values that can create a small graph.
7. Repeat steps 2 and 3 with the initial values found in 7.
This is the flowchart of my strategy.

Search all parameters x, y and z to reduce uncertainty factors.
1. Generate a derived graph randomly.
2. Calculate the diameter of the graph and ASPL.
3. Make a little change in voltage set.
4. Calculate the diameter and ASPL.
5. Judged whether the graph has become small.
6. If it is smaller, update the graph. If it is big, cancel the change.
7. It is determined whether or not convergence has occurred.
8. If it is not the end go back to 3.
Convergence

- **n=20000, d=11**
- **n=12000, d=7**
- **n=132000, d=8**

**ASPL**

- **ASPL=4.442422**
- **ASPL=5.169264**
- **ASPL=6.092304**

**ASPL**

<table>
<thead>
<tr>
<th>n</th>
<th>d</th>
<th>s</th>
<th>ASPL</th>
<th>std</th>
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<tbody>
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What I noticed (1/3)

• In voltage graph, Orders with many divisors give good results.
• I could not make a good graph at $n=4855$.
• However, $n=4860$ got a good graph.

As the number of divisors increases, the number of parameter sets also increases.
Therefore, we predict that good graphs are easy to calculate.
• Deleting one order lowers ASPL.

• At the time of $z = 1$, I could not get a good graph.
【Which voltage graph is better?】

• The voltage graph I used is $N = 2$ and $N = 1$.
• In some orders, $N = 4$ or $N = 8$ was better. (Specifically, 256 is $N = 4$. 20000 is $N = 8$.)
• There were other good voltage graphs, but it was not enough time to find out properly.
5. Source code
Source code

【URL】
https://github.com/Haruishimasato/voltage-graph/tree/Haruishimasato-programs

• I release the program on GitHub.

[2] Ibuki Kawamata (2017), Approximate evaluation and voltage assignment for order/degree problem

Graph Golf workshop, CANDAR 2017.