
Workshop in CANDAR'19, Nagasaki, Japan 2019-11-26
Graph Golf 2019
The Order/degree Problem Competition

Timetable

- 13:00-13:20 Introduction & Award ceremony
- 13:20-13:50 Keynote *by Kohta Nakashima*
- 13:50-14:15 Winner's talk *by Hajime Terao*
- 14:15-14:40 Winner's talk *by Team RK*
- 14:40-15:00 Organizer's talk *by Michihiro Koibuchi*

Introduction

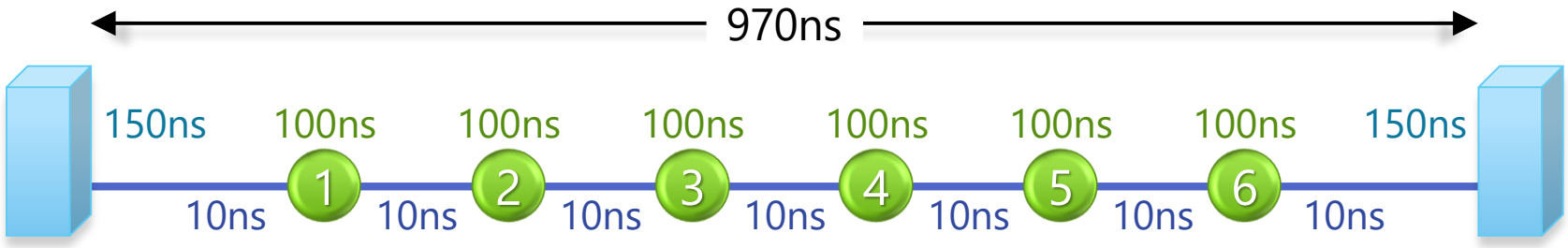
Graph Golf

- Online competition for the order/degree problem
- Aims at
  - Building a catalog of "best" graphs
  - Sharing knowledge for creating small-diameter graphs
  - Connecting graph-theoreticians and computer engineers
- Past workshops
  - CANDAR '15, '16, '17, '18
  - Intl. Symp. on Networks-on-Chip (NOCS 2016)
  - FIT 2016 (Japanese)
- NII News Release (Japanese)
  - https://www.nii.ac.jp/news/release/

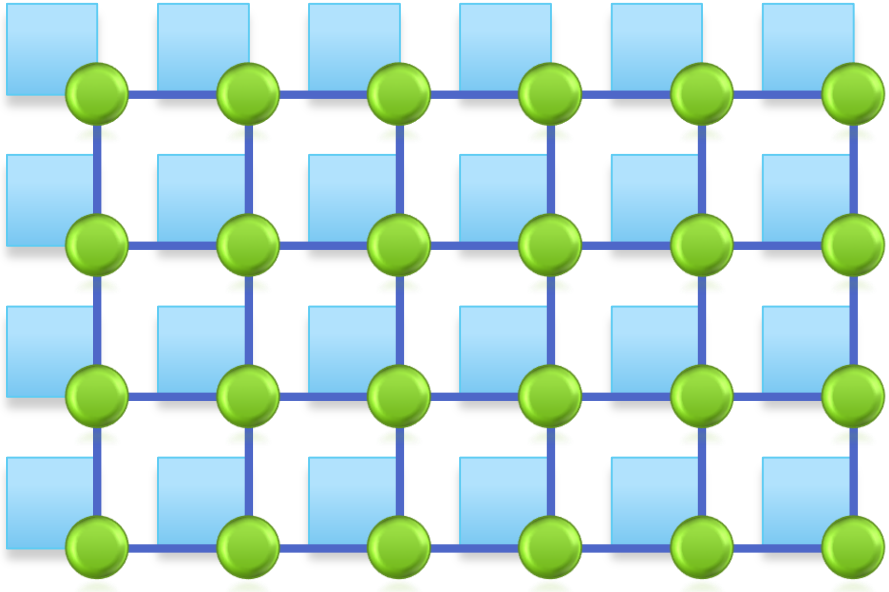
Winners of the 2015 competition

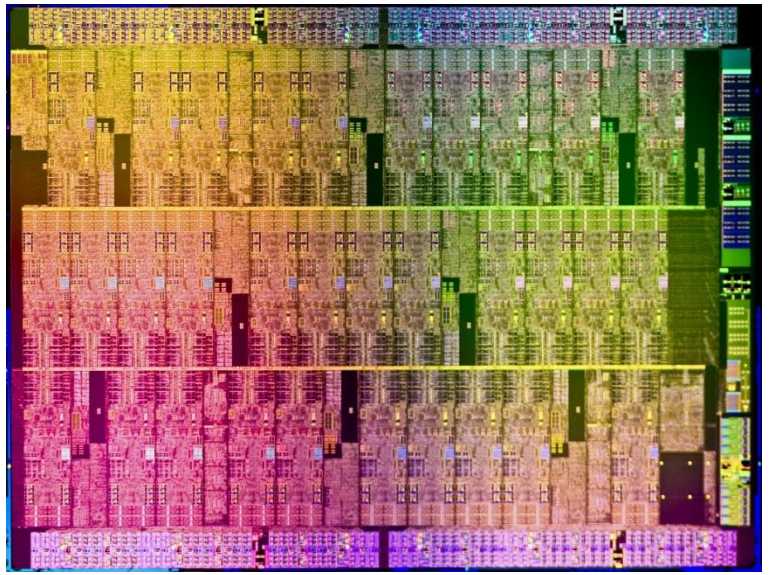
Background

- Low-latency networks are desired in supercomputers
  - 1μs for 3M cores ≒ **7 hops for 100k nodes**



- On-chip networks will soon face the same problem





https://www.extremetech.com/extreme/171678-intel-unveils-72-core-x86-knights-landing-cpu-for-exascale-supercomputing

Degree/Diameter Problem (DDP)

- Given a diameter k and a degree d, find a graph with the largest order n.
  - Known solutions are listed in the *Combinatorics Wiki*

http://combinatoricswiki.org/wiki/The_Degree_Diameter_Problem_for_General_Graphs

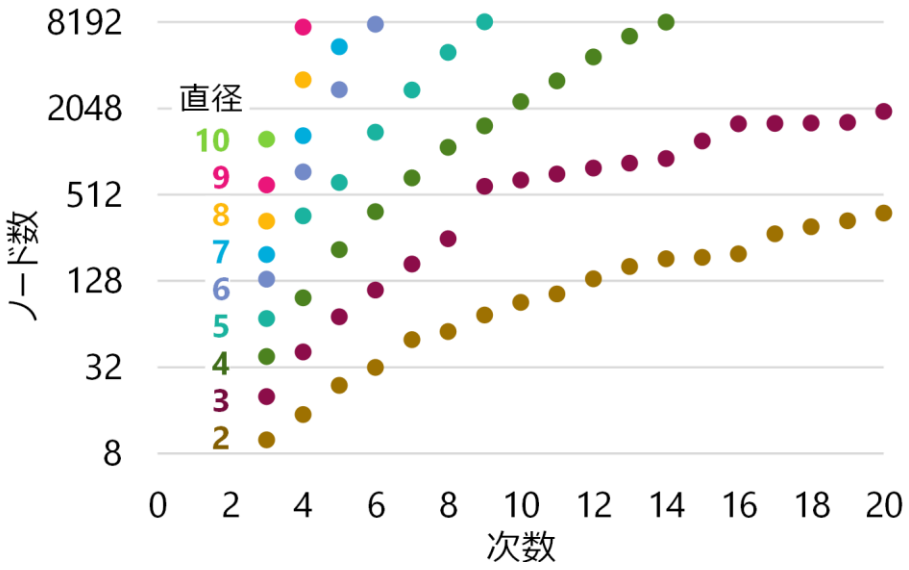


Table of the orders of the largest known graphs for the undirected degree diameter problem

$d \backslash k$	2	3	4	5	6	7	8	9	10
3	10	20	38	70	132	196	336	600	1 250
4	15	41	98	364	740	1 320	3 243	7 575	17 703
5	24	72	212	624	2 772	5 516	17 030	57 840	187 056
6	32	111	390	1 404	7 917	19 383	76 461	331 387	1 253 615
7	50	168	672	2 756	11 988	52 768	249 660	1 223 050	6 007 230
8	57	253	1 100	5 060	39 672	131 137	734 820	4 243 100	24 897 161
9	74	585	1 550	8 268	75 893	279 616	1 697 688	12 123 288	65 866 350
10	91	650	2 286	13 140	134 690	583 083	4 293 452	27 997 191	201 038 922
11	104	715	3 200	19 500	156 864	1 001 268	7 442 328	72 933 102	600 380 000
12	133	786	4 680	29 470	359 772	1 999 500	15 924 326	158 158 875	1 506 252 500
13	162	851	6 560	40 260	531 440	3 322 080	29 927 790	249 155 760	3 077 200 700
14	183	916	8 200	57 837	816 294	6 200 460	55 913 932	600 123 780	7 041 746 081
15	187	1 215	11 712	76 518	1 417 248	8 599 986	90 001 236	1 171 998 164	10 012 349 898
16	200	1 600	14 640	132 496	1 771 560	14 882 658	140 559 416	2 025 125 476	12 951 451 931
17	274	1 610	19 040	133 144	3 217 872	18 495 162	220 990 700	3 372 648 954	15 317 070 720
18	307	1 620	23 800	171 828	4 022 340	26 515 120	323 037 476	5 768 971 167	16 659 077 632
19	338	1 638	23 970	221 676	4 024 707	39 123 116	501 001 000	8 855 580 344	18 155 097 232
20	381	1 958	34 952	281 820	8 947 848	55 625 185	762 374 779	12 951 451 931	78 186 295 824

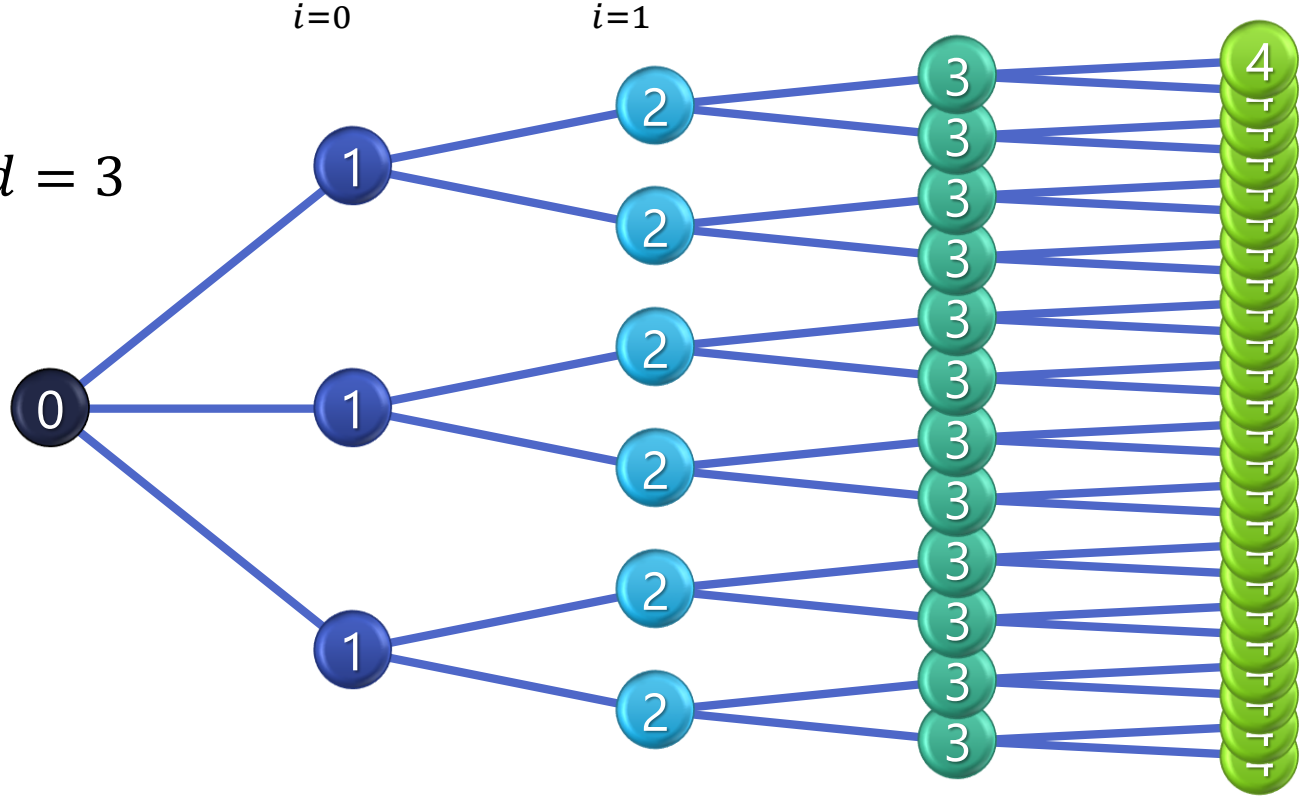
- Solutions given are not very useful for interconnects
  - Because these orders don't fit hardware design

Moore Bound for DDP

- Given diameter k and degree d, the upper bound of the order n is

$$N_{d,k} = \sum_{i=0}^{k} n_i = 1 + d\sum_{i=1}^{k} (d-1)^{i-1}$$

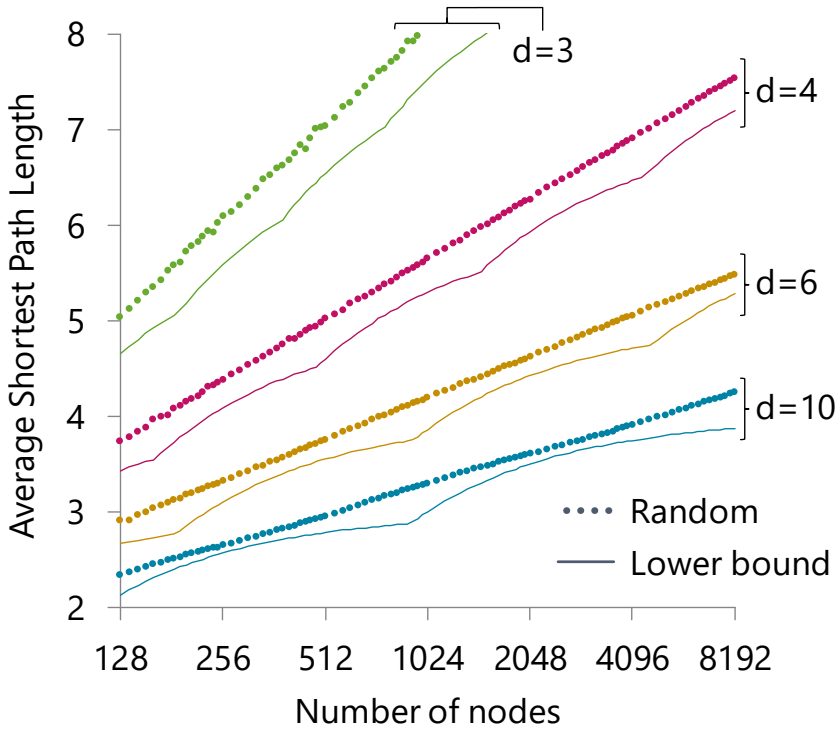
- Example for $d = 3$



Order n_k =	1	3	6	12	24
U.B. $N_{d,k}$ =	1	4	10	22	46

Order/Degree Problem (ODP)

- Given an order n and a degree d, find the graph with the smallest diameter k.
  - Among those with the same diameter, find the one with the smallest ASPL l.
- Gives useful solutions to design interconnects
  - However, theoreticians are not interested in ODP (why?)
- There can be better graphs than the random graphs



Moore Bound for ODP

- The lower bound $K_{n,d}$ of diameter k for order n and degree d

$$K_{n,d} = \begin{cases} \left\lceil \frac{n-1}{2} \right\rceil & \text{if } d = 2 \\ \left\lceil \log_{d-1}\left(\frac{(n-1)(d-2)}{d} + 1 \right) \right\rceil & \text{if } d > 2 \end{cases}$$

- The lower bound $L_{n,d}$ of ASPL l for order n and degree d

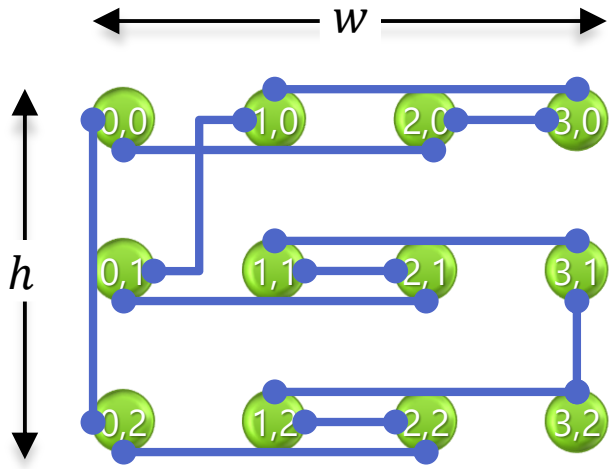
$$L_{n,d} = \begin{cases} 1 & \text{if } K_{n,d} = 1 \\ \frac{\sum_{i=1}^{K_{n,d}-1} id(d-1)^{i-1} + K_{n,d}\left(n - 1 - \sum_{i=1}^{K_{n,d}-1} d(d-1)^{i-1}\right)}{n-1} & \text{if } K_{n,d} \geq 2 \end{cases}$$

- ASPL gap p

$$p = \frac{l - L_{n,d}}{L_{n,d}}$$

ODP on Grid Graphs

- Given a degree d, find the graph with the smallest diameter k on a $w \times h$ grid, while keeping the lengths of each edge $\leq r$ in a Manhattan distance
  - Example: $w \times h = 4 \times 3,\ d = 2,\ r = 2$



- Either the Moore Bound or the edge length limitation dominates the diameter

Award Rules

Widest Improvement Award

- Who find the largest number of "best" solutions
- The best solution means a graph with the smallest diameter, and with the smallest ASPL among those with the same diameter, for each order/degree pair.

Deepest Improvement Award

- Who achieve the smallest ASPL gap over all the order/degree pairs

http://freedesignfile.com/9420-trophy-cup-and-medals-vector-set-05

2019 Problems & Rankings

The 2019 Problems

- General Graph Category

n	d	
50	4	For practice
512	4	Memory cube
512	6	Memory cube
1024	4	Memory cube
1726	30	Piz Daint
4855	15	Trinity
9344	6	Titan
65536	6	BluleGene/L
100000	8	Exascale roadmap
1000000	16	Exascale roadmap
1000000	32	Exascale roadmap

- Grid Graph Category

$w \times h$	d	r	
5×5	4	2	For practice
5×15	4	3	For practice
10×10	4	2	Balanced
10×10	6	3	Balanced
10×10	8	4	Balanced
20×20	4	2	4×
20×20	6	3	4×
20×20	8	4	4×
100×100	4	2	100×
100×100	6	3	100×
100×100	8	4	100×

Widest Improvement Rankings

- General Graph

Rank	Authors	#Best solutions
1	Hajime Terao	5
2	Team RK	4
3	Teruaki Kitasuka	3
4	EvbCFfp1XB	1
4	Tomas Rokicki	1

- Grid Graph

Rank	Authors	#Best solutions
1	Team RK	11
2	EvbCFfp1XB	1
2	Teruaki Kitasuka	1

Deepest Improvement Rankings

- General Graph

Rank	Authors	ASPL gap
1	Hajime Terao	0.01833711186121878
2	Team RK	0.0264187866927594 3
3	Tomas Rokicki	0.04897959183673484
3	EvbCFfp1XB	0.04897959183673484
5	Teruaki Kitasuka	0.10990648883547927

- Grid Graph

Rank	Authors	ASPL gap
1	Team RK	0.009495549603009579
2	Teruaki Kitasuka	0.05333333336666566
2	EvbCFfp1XB	0.05333333336666566

Remarkable Achievements

- Diameter gap = 0 is acheieved at all the featured grid graphs and 4 general graphs:

n	d	k	l	ASPL gap	Found by
512	4	6	4.77054794520548	0.17172211350294	Team RK
1726	30	3	2.47920667707861	0.01833711186122	Hajime Terao
4855	15	4	3.40842317610371	0.10990648883548	Teruaki Kitasuka
1000000	16	6	5.35418235418235	0.28402328402328	Teruaki Kitasuka

where the random graph's diameter > 0

- Tomas Rokicki submitted 1,217 *non-featred* graphs!
  - Generated by hill-climbing method

Award Ceremony

General Graph Widest and Deepest Improvement Award

Hajime Terao

The University of
Electro-Communications

Grid Graph Widest and Deepest Improvement Award

Team RK

Masahiro Nakao*

Maaki Sakai**

Yoshiko Hanada**

* RIKEN R-CCS

** Kansai University

Congratulations!!

Graph Golf 2019
Organizing Committee