Meta-Level Abduction

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Meta-level abduction
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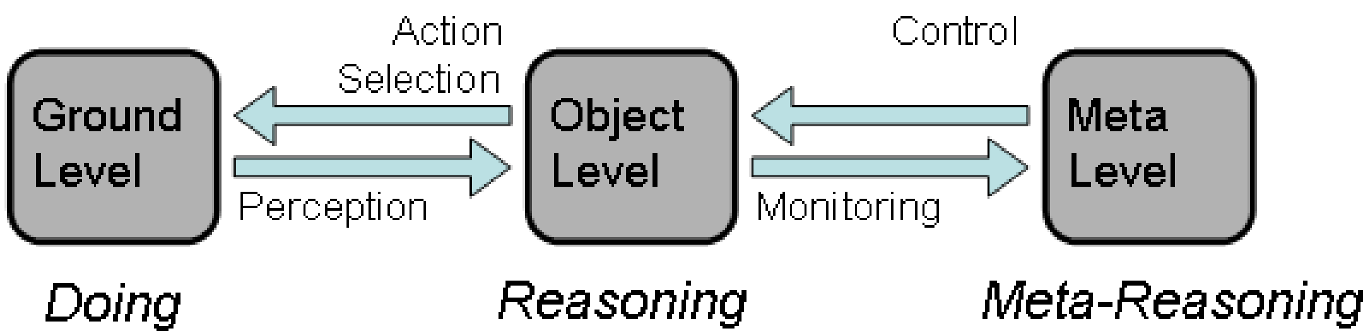
The items 1—6 will appear in: Inoue, K., "Meta-level abduction", *IfCoLog Journal of Logics and their Applications*, Special Issue of "Frontiers of Abduction" (2015).

Meta-reasoning



- Pereira, Bundy, Russel, et al.
  - The meta-level control of the reasoning process itself.
  - The introspective monitoring of the reasoning process at the object level.
- Kowalski initiated *meta-programming* in logic programming, e.g., [Bowen & Kowalski, 1982]

“Vanilla” meta-interpreter (Hill & Lloyd)

$solve(true).$

$solve(A \,\&\, B) \leftarrow solve(A) \,\&\, solve(B).$

$solve(\neg A) \leftarrow \neg solve(A).$

$solve(A) \leftarrow rule(A \leftarrow B) \,\&\, solve(B).$

- In general, any inference rule can be expressed in such a meta-rule, e.g.,

  $solve(A \leftarrow B) \leftarrow solve(A \leftarrow C) \,\&\, solve(C \leftarrow B).$

  $solve(\neg A) \leftarrow solve(B \leftarrow A) \,\&\, solve(\neg B).$

- All constructs with predicates solve, clause, demo are atoms.
- Those meta-level axioms are used for *deduction* only.

Abductive inference

- Abduction augments *sufficient conditions* missing in the premises (background knowledge) to enable a derivation (proof) of the given observation (goal).
- This inference *fills the gap* in a proof of the observation from the premises.
- Inferred sufficient conditions are called **hypotheses** or **explanations**. Often, they are called **explanans**. The observation is called the **explanundum**.
- A hypothesis can be any formula, e.g., a set (conjunction) of atoms/literals/rules.

Three modes of inference

(C.S. Peirce)

deduction

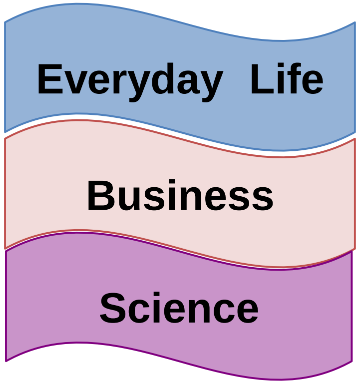
$$\frac{A \rightarrow C \qquad A}{C}$$

induction

$$\frac{A \qquad C}{A \rightarrow C}$$

abduction

$$\frac{A \rightarrow C \qquad C}{A}$$



Meta-level abduction

- *Abduction is performed on meta-level axioms.*
- For example, given a meta-theory:

  $solve(A) \leftarrow rule(A \leftarrow B) \land solve(B),$

  $solve(B),$

  and a meta-goal:

  $solve(A),$

  we can abduce

  $rule(A \leftarrow B).$
- In this example, we can realize *rule abduction*.
- Yet, this is an ordinary abduction since it abduce atoms.

Meta-level abduction (Inoue *et al.*, 2009-)

- A method to **abduce rules**, enabling to infer hypotheses
  - explaining *empirical rules* by means of *hidden rules*,
  - representing *multiple missing links / causal relations*,
  - simultaneously accounting for *multiple observations*,
  - containing *unknown nodes* as **new predicates**.
  - A new way of induction based on **full-clausal abduction**.
- Combination of *rule abduction* and *fact abduction* is possible by way of **conditional query answering**.
- First motivating examples were from *cognitive modeling* for improving the skill of music playing (ILP '09, *LNAI* 5989)
- The method has been applied to ***scientific discovery*** *from* ***network data***, e.g., genetic/biochemical pathways (ILP '10, *LNAI* 6489; *Machine Learning* 91, 2013; FMMB '14, *LNBI* 8738).

Patterns of Abduction (Schurz, *Synthese*, 2008)

Kind of abduction	Explanandum	Explanans
Factual abduction	Single facts	New facts
— Observable-fact abduction	Single facts	Factual reasons
— 1st-order existential abduction	Single facts	Facts with new unknown individuals
— Unobservable-fact abduction	Single facts	Unobservable facts
Law abduction	Empirical laws	New laws
Theoretical-model abduction	General empirical phenomena	New theoretical models
2nd-order existential abduction	General empirical phenomena	New laws with new concepts
— Micro-part abduction	General empirical phenomena	Microscopic compositions
— Analogical abduction	General empirical phenomena	New laws with analogical concepts
— Hypothetical cause abduction	General empirical phenomena	Hidden (unobservable) causes

Factual Meta-Level Abduction

Kind of abduction	Explanandum	Explanans
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— Unobservable-fact abduction	Single facts	Unobservable facts
Law abduction	Empirical laws	New laws
Theoretical-model abduction	General empirical	New theoretical models
2nd-order existential abduction	phenomena	New laws with new concepts
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Meta-level

A simple logic of causality

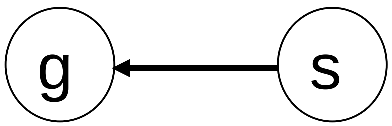
- To express relations between events, we use **causal chains**.
- Causality can be represented in first-order predicate logic.
- Two meta-predicates:
  1. ***linked*(X,Y)**: X is *directly caused by* Y.
  2. ***caused*(X,Y)**: There is a *causal chain* from Y to X.

- Basic axioms:

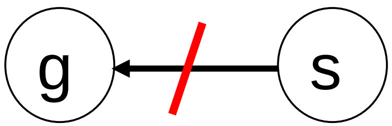
$$\mathit{caused}(X,Y) \leftarrow \mathit{linked}(X,Y).$$

$$\mathit{caused}(X,Y) \leftarrow \mathit{linked}(X,Z) \land \mathit{caused}(Z,Y).$$

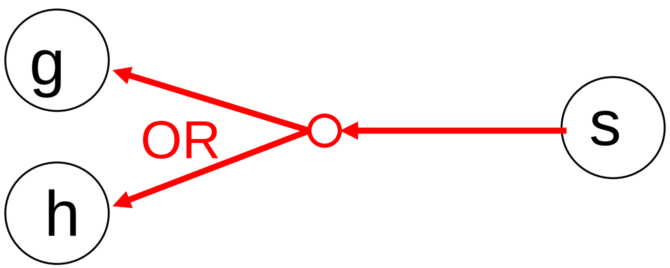
Representing logical connectives



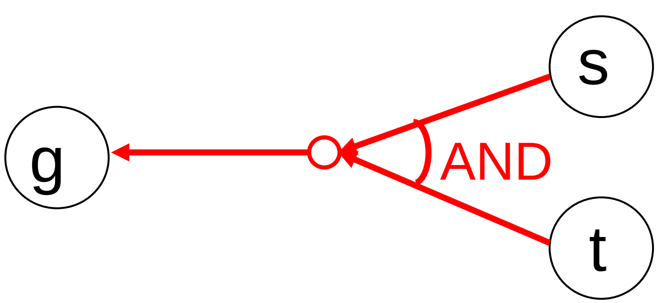
linked(g, s).



$\neg$*linked(g, s).*



linked(g, s) $\vee$ *linked(h, s)*



linked(g, s) $\vee$ *linked(g, t)*

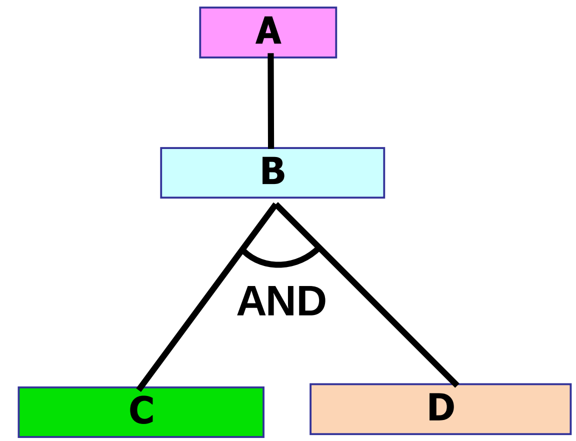
Object and meta level representation

- **<u>Object domain (object level)</u>**

  A ← B.

  B ← C ∧ D.

- Each *rule* in the object level is represented as a *fact* in the meta level.
- Each *literal* in the object level is represented as a *term* in the meta level.

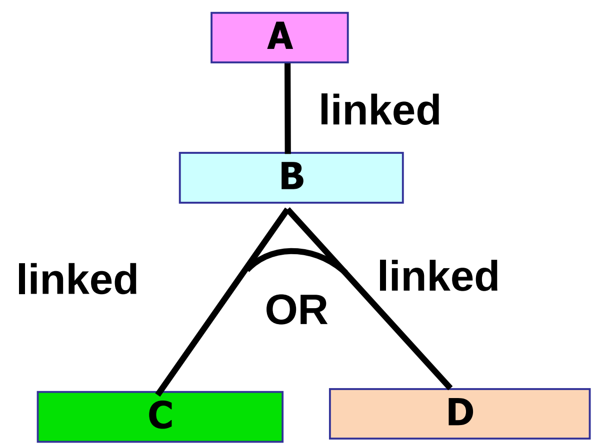


- **<u>Causal relations (meta level)</u>**

  linked(A, B).

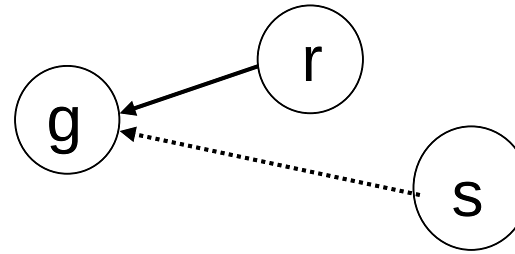
  linked(B, C) ∨ linked(B, D).

- *Rule abduction* in the object level is realized by abducing literals of the form *linked(_,_)* at the meta level.



Formalizing rule abduction

- *g*: a **goal**, *s*: an **input**, *r*: a (hidden) node



B: *linked(g, r).*
$\neg$*linked(g, s).*

That is, *g* is directly caused by *r*, but *g* is ***not*** directly caused by *s*.

- *g* is not directly caused by *s*, but we know that there is a causal chain to *g* from *s*.

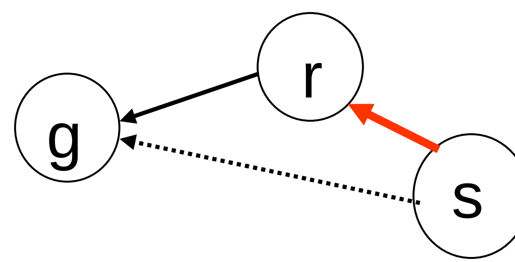
  This is given by an observation:

G: *caused(g, s).*

- SOLAR computes a hypothesis

H: *linked(r, s)*,

given the abducibles {*linked(_,_)*}.



Patterns of Abduction (Schurz, *Synthese*, 2008)

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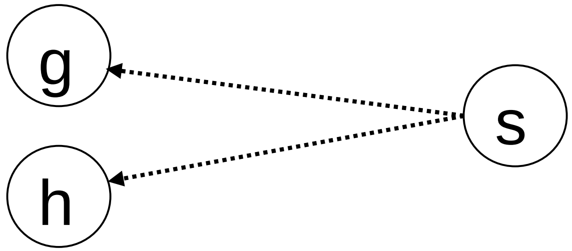
Meta-level

Object level

Node introduction = Predicate invention

- *g, h* : goal nodes, *s*: an input node.

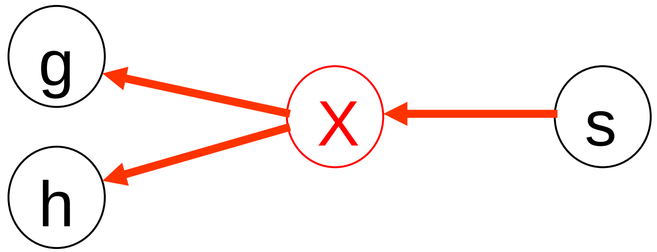


B: $\neg$ *linked(g, s).*
$\neg$ *linked(h, s).*

That is, there are no direct causal relations from *s* to *g* and from *h* to *s*, but there are causal chains as the observations:

G: *caused(g, s)* $\wedge$ *caused(h, s)*.

- Given the abducibles {*linked(_,_)*}, SOLAR generates a hypothesis **H:**



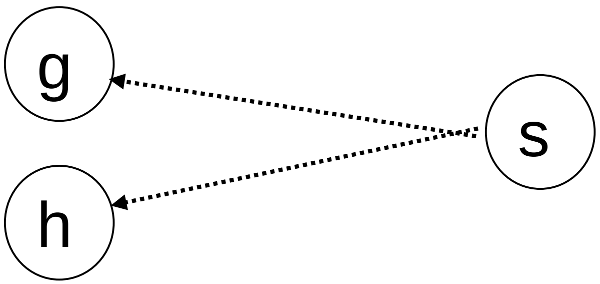
$\exists X.$ (*linked(g, X)* $\wedge$ *linked(h, X)* $\wedge$ *linked(X, s)*).

- Variable *X* represents a newly introduced node, which corresponds to **predicate invention** or **object invention**.

Representing different structures

B: $\neg$ *linked(g, s).*
$\neg$ *linked(h, s).*

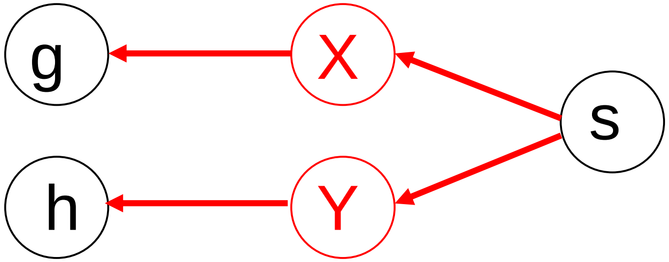
G: *caused(g, s).*
caused(h, s).



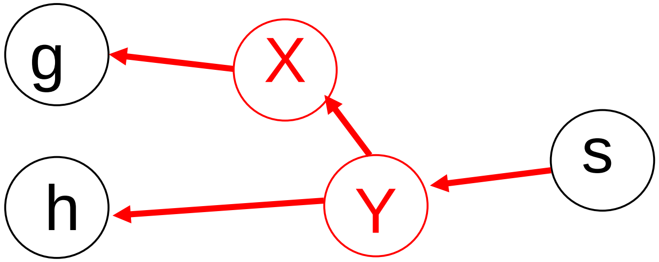
Abducibles: {*linked(_,_)*}.

H with 2 intermediate nodes:

$\exists X \exists Y.$ (*linked*(*g*, *X*) $\wedge$ *linked*(*h*, *Y*) $\wedge$ *linked*(*X*, *s*) $\wedge$ *linked*(*Y*, *s*)).



$\exists X \exists Y.$ (*linked*(*g*, *X*) $\wedge$ *linked*(*h*, *Y*) $\wedge$ *linked*(*X*, *Y*) $\wedge$ *linked*(*Y*, *s*)).



Correctness of meta-level abduction

Proposition: Let $\lambda(B)$ be the theory obtained by replacing every *linked(g, s)* appearing in *B* with the formula $(g \leftarrow s)$.

$$B \cup \mathit{Meta\text{-}Axioms} \models (\mathit{caused}(g, s_1) \vee \cdots \vee \mathit{caused}(g, s_n))$$

$$\text{iff } \lambda(B) \models (g \leftarrow s_1 \wedge \cdots \wedge s_n).$$

Theorem: Observation: $O = (\mathit{caused}(g, s_1) \vee \cdots \vee \mathit{caused}(g, s_n))$.

H is an abductive explanation of O with respect to B and $\Gamma_M = \{\mathit{linked}(\_,\_)\}$ *iff* $\lambda(H)$ *is a hypothesis such that*

- $\lambda(B) \cup \lambda(H) \models (g \leftarrow s_1 \wedge \cdots \wedge s_n)$, *and*
- $\lambda(B) \cup \lambda(H)$ *is consistent.*

Implementation

- Although the idea of meta-level abduction is simple, its implementation requires an abductive procedure for first-order full clausal theories.
- Currently, SOLAR (Nabeshima *et al*., 2010) is only such a state-of-the-art procedure.
- There will be a possibility to use logic programming and answer set programming.

Application to “knack” discovery

B: $linked$(inc_sound, bow_close_to_the_bridge).

$linked$(bow_close_to_the_bridge, stable_bow_movement) $\vee$

$linked$(bow_close_to_the_bridge, smooth_bow_direction_change).

$linked$(smooth_bow_direction_change, flexible_wrist).

$\leftarrow$ $linked$(inc_sound, keep_arm_close).

$\leftarrow$ $linked$(stable_bow_movement, keep_arm_close).

$\leftarrow$ $linked$(smooth_bow_direction_change, keep_arm_close).

G: *caused*(inc_sound, keep_arm_close).

- SOLAR generates 52 hypotheses (maximum search depth: 15, maximum length of produced clauses: 5). One of them is:

$\exists X$. (*linked*(stable_bow_movement, X) $\wedge$ *linked*(flexible_wrist, X)

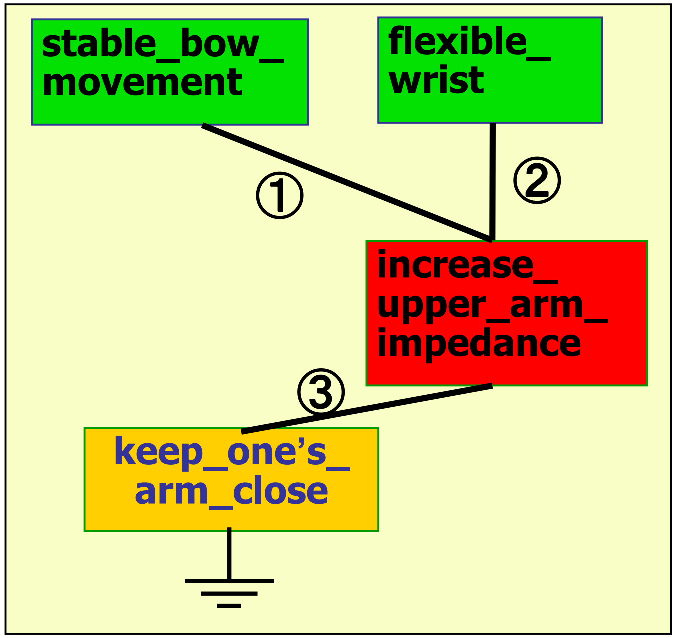
$\wedge$ *linked*(X, keep_arm_close)).

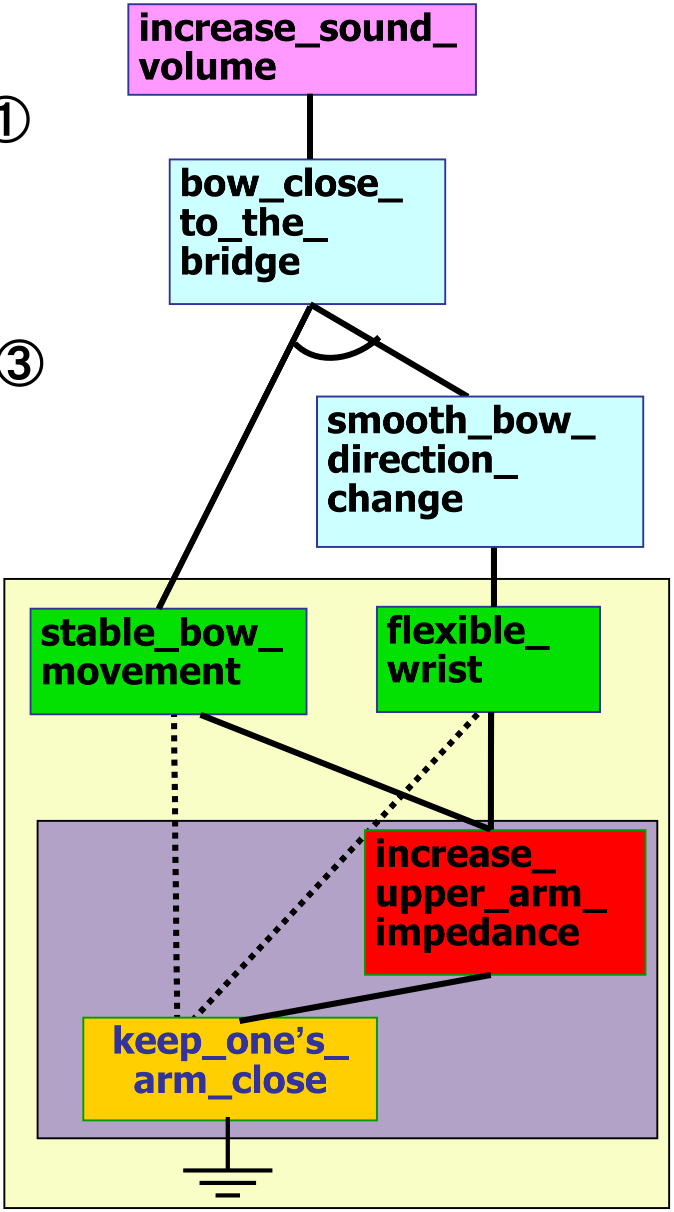
The obtained hypothesis

H: $\exists X.\ (\mathit{linked}(\text{stable\_bow\_movement}, X)$ ①

$\wedge\ \mathit{linked}(\text{flexible\_wrist}, X)$ ②

$\wedge\ \mathit{linked}(X, \text{keep\_arm\_close})\ ).$ ③





Abducing facts

New axioms:

$caused(X, X) \leftarrow abd(X).$ *% for abducibles*

$caused(X, Y) \leftarrow linked(X, Y).$

$caused(X, Y) \leftarrow linked(X, Z) \land caused(Z, Y).$

The top clause:

$\leftarrow caused(g, X) \land abd(X).$

Note: *abd* plays the role of an *answer predicate.*

An integrity constraint that *p* and *q cannot hold simultaneously:*

$\leftarrow caused(p, X) \land caused(q, Y) \land abd(X) \land abd(Y).$

Abducing facts and rules

- Abducing facts is nothing but **answer extraction**.
- Abducing facts and rules is then **conditional query answering**.

Patterns of Abduction (Schurz, *Synthese*, 2008)

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Meta-level

Object level

General Logic of Causality (GLC)

- Now we do not consider any particular interpretation of the meta-predicate *linked* so that its semantics is left open.
- This is desirable, since $caused(g, s)$ cannot be interpreted as the material implication $(g \leftarrow s)$ in general. Instead, the meaning of causality at the object level is now abstracted away.
- In particular, the correspondence such that $(caused(g, s_1) \vee \cdots \vee caused(g, s_n)) \Leftrightarrow (g \leftarrow s_1 \wedge \cdots \wedge s_n)$ is abandoned.
- In GLC, a causal network is given as a set of atoms of the form *linked*(F,G), where F and G are any Boolean formulas. Similarly, atoms of the form *caused*(F,G) are allowed with any F and G.
- This logic, called GLC, is defined by *structural inference rules*.
- Here, we denote the meta-atoms $linked(g, s)$ and $caused(g, s)$ as the meta-relations $(s \rhd g)$ and $(s \vDash g)$, respectively.

Postulates for GLC

- (Reflection) $\dfrac{\alpha \triangleright \psi}{\alpha \vDash \psi}$
- (Weakening) $\dfrac{\alpha \vDash \beta \qquad \beta \succcurlyeq \psi}{\alpha \vDash \psi}$
- (Strengthening) $\dfrac{\alpha \succcurlyeq \beta \qquad \beta \vDash \psi}{\alpha \vDash \psi}$
- (And) $\dfrac{\alpha \vDash \varphi \qquad \alpha \vDash \psi}{\alpha \vDash \varphi \wedge \psi}$
- (Cut) $\dfrac{\alpha \vDash \beta \qquad \alpha \wedge \beta \vDash \psi}{\alpha \vDash \psi}$
- (Or) $\dfrac{\alpha \vDash \psi \qquad \beta \vDash \psi}{\alpha \vee \beta \vDash \psi}$

Note: $\succcurlyeq$ is a classical Tarski consequence operator; $X \succcurlyeq Y$ iff X entails Y.

GLC meta-program

$caused(X, Y) \leftarrow linked(X, Y).$

$caused(X, Y) \leftarrow (X \preceq Z) \wedge caused(Z, Y).$

$caused(X, Y) \leftarrow caused(X, Z) \wedge (Z \preceq Y).$

$caused(X \wedge Y, Z) \leftarrow caused(X, Z) \wedge caused(Y, Z).$

$caused(X, Y) \leftarrow caused(X, Y \wedge Z) \wedge caused(Z, Y).$

$caused(X, Y \vee Z) \leftarrow caused(X, Y) \wedge caused(X, Z).$

Note: $\preccurlyeq$ is a classical consequence operator;

$X \preccurlyeq Y$ iff $Y \succcurlyeq X$ iff Y entails X, e.g., $s \preccurlyeq s \wedge t$

GLC is more general than SLC

- **Proposition.** *Suppose a theory T for SLC, and let T′ be a theory for GLC obtained from T by replacing every disjunctive fact of the form* ($linked(p, q_1) \vee \cdots \vee linked(p, q_m)$) *with a fact of the form* $linked(p, q_1 \wedge \cdots \wedge q_m)$. *If* $G = (caused(g, s_1) \vee \cdots \vee caused(g, s_n))$ *is derived from T in SLC, then* $G' = caused(g, s_1 \wedge \cdots \wedge s_n)$ *is derived from T′ in GLC.*

- **Proof (sketch):** Reflection in GLC corresponds to the first meta-axiom in SLC. The second meta-axiom in SLC is obtained by Transitivity (which is a consequence of Strengthening and Cut) and Reflection.

  The case inference in SLC can be simulated in GLC as follows. Suppose ($linked(q, p_1) \vee linked(q, p_2)$) in SLC. From $caused(p_1, s_1)$ and $caused(p_2, s_2)$, ($caused(q, s_1) \vee caused(q, s_2)$) is derived in SLC. The corresponding inference can be obtained in GLC as:

$$
\dfrac{\dfrac{\dfrac{s_1 \models p_1}{s_1 \wedge s_2 \models p_1} \qquad \dfrac{s_2 \models p_2}{s_1 \wedge s_2 \models p_2}}{s_1 \wedge s_2 \models p_1 \wedge p_2} \qquad \dfrac{p_1 \wedge p_2 \rhd q}{p_1 \wedge p_2 \models q}}{s_1 \wedge s_2 \models q}
$$

Transitivity in GLC

- Strengthening and Cut imply the transitivity:

- (Transitivity) $\dfrac{\alpha \vDash \beta \qquad \beta \vDash \psi}{\alpha \vDash \psi}$

- There is a controversy on the validness of transitivity, e.g., Hitchcock, *J. Philos.*, 98 (2001). Also, McDermott (1995), Pearl (2000, 2009), Halpern & Pearl (2000), Woodward (2004), etc.
- But, we can interpret the causality here simply as an *influence*. Moreover, many anomaly cases can be avoided if we carefully model the causal theory.

Monotonicity in GLC

- (Left) Strengthening is also called Monotonicity (Kraus, Lehmann & Magidor, 1990):

$$\text{(Strengthening)} \qquad \frac{\alpha \succcurlyeq \beta \qquad \beta \vDash \psi}{\alpha \vDash \psi}$$

- Monotonicity apparently fails to hold in *nonmonotonic logics*.
- **Example**. *"Striking a match causes it to light"* obviously does not imply "*Putting a match in water and then striking it causes it to light*." (Bochman & Gabbay, 2012)

Networks with positive and negative causal links (Inoue, Doncescu & Nabeshima, 2010-2013)

- Consider networks with both positive and negative causal effects.
- In biology, such networks appear in gene regulatory/transcription systems, signaling networks, and metabolic pathways.
- Two types of direct causal relations: **triggered** and **inhibited**.
- ***triggered(g, t)*** : a positive direct cause (t is a trigger of g)

$$g \longleftarrow t$$

in a causal graph, whose meaning is ($g \Leftarrow t$) in the object level, where $\Leftarrow$ means that the causation appears *if it is not prevented*.

- ***inhibited(g, s)*** : a negative direct cause (s is an *inhibitor* of g)

$$g \dashv s$$

in a causal graph, whose meaning is ($\neg g \Leftarrow s$) in the object level.

Alternating axioms for causality

- **Causal chains** have two kinds too:
  1. **promoted(X,Y)**: X is *positively caused* by Y.
  2. **suppressed(X,Y)**: X is *negatively caused* by Y.

$$caused(X,Y) \leftarrow linked(X,Y).$$
$$caused(X,Y) \leftarrow linked(X,Z) \land caused(Z,Y).$$

$$promoted(X, Y) \leftarrow triggered(X, Y).$$
$$promoted(X, Y) \leftarrow triggered(X,Z) \land promoted(Z, Y).$$
$$promoted(X, Y) \leftarrow inhibited(X,Z) \land suppressed(Z, Y).$$
$$suppressed(X, Y) \leftarrow inhibited(X, Y).$$
$$suppressed(X, Y) \leftarrow inhibited(X,Z) \land promoted(Z, Y).$$
$$suppressed(X, Y) \leftarrow triggered(X,Z) \land suppressed(Z, Y).$$
$$\leftarrow promoted(X, Y) \land suppressed(X,Y).$$

Monotonic property

- Meta-level abduction is defined for an observation

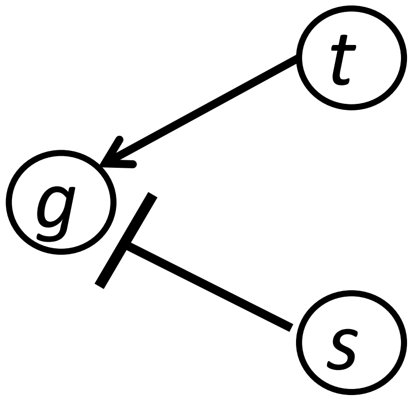
  promoted(g, s) or *suppressed(g, s)*

  with the abducibles

  Γ = { *triggered(_,_), inhibited(_,_)* }.
- Given positive and negative observations, both positive and negative direct causes are abduced and new nodes are produced when necessary.

- **Proposition**: *For any suppression (resp. promotion) for g from s, there is a causal chain P from s to g such that there exist an odd number of (resp. an even number of) direct inhibitions in P.*

Antagonistic factors



triggered(g, t)

inhibited(g, s)

Suppose that both the trigger *t* and the suppressor *s* are activated. Is *g* promoted or suppressed? Intuitions are:

(1) If *t* works and *s* does not work, then *g* is promoted by *t*;

(2) If *s* works and *t* does not work, then *g* is suppressed by *s*;

(3) If both *t* and *s* work, then *g* is suppressed by *s*.

Namely, *an inhibitor is preferred to a trigger*.

Axioms with defaults

- Causal chains should have *nonmonotonic* effects.

$promoted(X, Y) \leftarrow triggered(X, Y) \wedge no\_inhibitor(X).$

$promoted(X, Y) \leftarrow triggered(X, Z) \wedge no\_inhibitor(X) \wedge promoted(Z, Y).$

$promoted(X, Y) \leftarrow inhibited(X, Z) \wedge suppressed(Z, Y).$

$suppressed(X, Y) \leftarrow inhibited(X, Y).$

$suppressed(X, Y) \leftarrow inhibited(X, Z) \wedge promoted(Z, Y).$

$suppressed(X, Y) \leftarrow triggered(X, Z) \wedge no\_inhibitor(X) \wedge suppressed(Z, Y).$

$\leftarrow promoted(X, Y) \wedge suppressed(X, Y).$

no_inhibitor(_) : treated as a *default*, which can be assumed during inference unless contradiction occurs.

Abduction with defaults

- For default assumptions of the form *no_inhibitor(_), their negations* are skipped in SOLAR by putting them in the production field.
- Membership of a clause *C* in an extension of a default theory is guaranteed for each obtained consequence of the form

  $$C \leftarrow no\_inhibitor(t1) \wedge no\_inhibitor(t2) \wedge \cdots$$

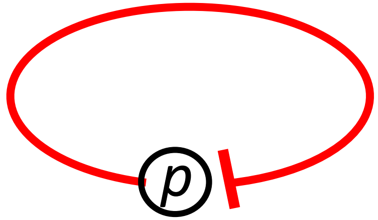
  [Inoue *et al*., 2004, 2006].

Correspondence between object-level inference and meta-level consequence finding

object-level inference	top clause in SOLAR *	production field
proving rules	$\neg caused(g, s)$	none
abducing facts	$\neg caused(g,X) \lor ans(X)$	$ans(\_)$
predicting facts	$\neg caused(X, s) \lor ans(X)$	$ans(\_)$
predicting rules	none	$promoted(\_,\_)$, $suppressed(\_,\_)$
abducing rules	$\neg caused(g, s)$	$\neg triggered(\_,\_)$, $\neg inhibited(\_,\_)$
abducing rules and facts	$\neg caused(g,X) \lor \neg abd(X)$	$\neg triggered(\_,\_)$, $\neg inhibited(\_,\_)$, $ans(\_)$
predicting conditional facts	$\neg caused(X, s) \lor ans(X)$	$\neg triggered(\_,\_)$, $\neg inhibited(\_,\_)$, $ans(\_)$
predicting conditional rules	none	$\neg triggered(\_,\_)$, $\neg inhibited(\_,\_)$, $promoted(\_,\_)$, $suppressed(\_,\_)$

* $\neg caused(X,Y)$ is instantiated by either $\neg promoted(X,Y)$ or $\neg suppressed(X,Y)$.

Consistency of meta-level abduction

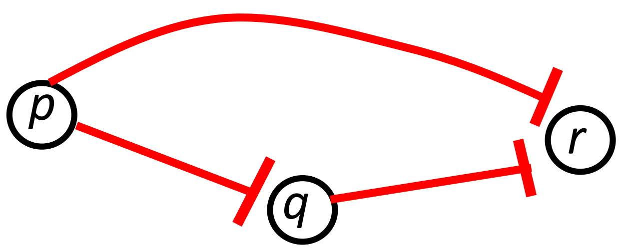


B_0 = { *inhibited*(p, p) }.

B_0 U Meta-Axioms is inconsistent.

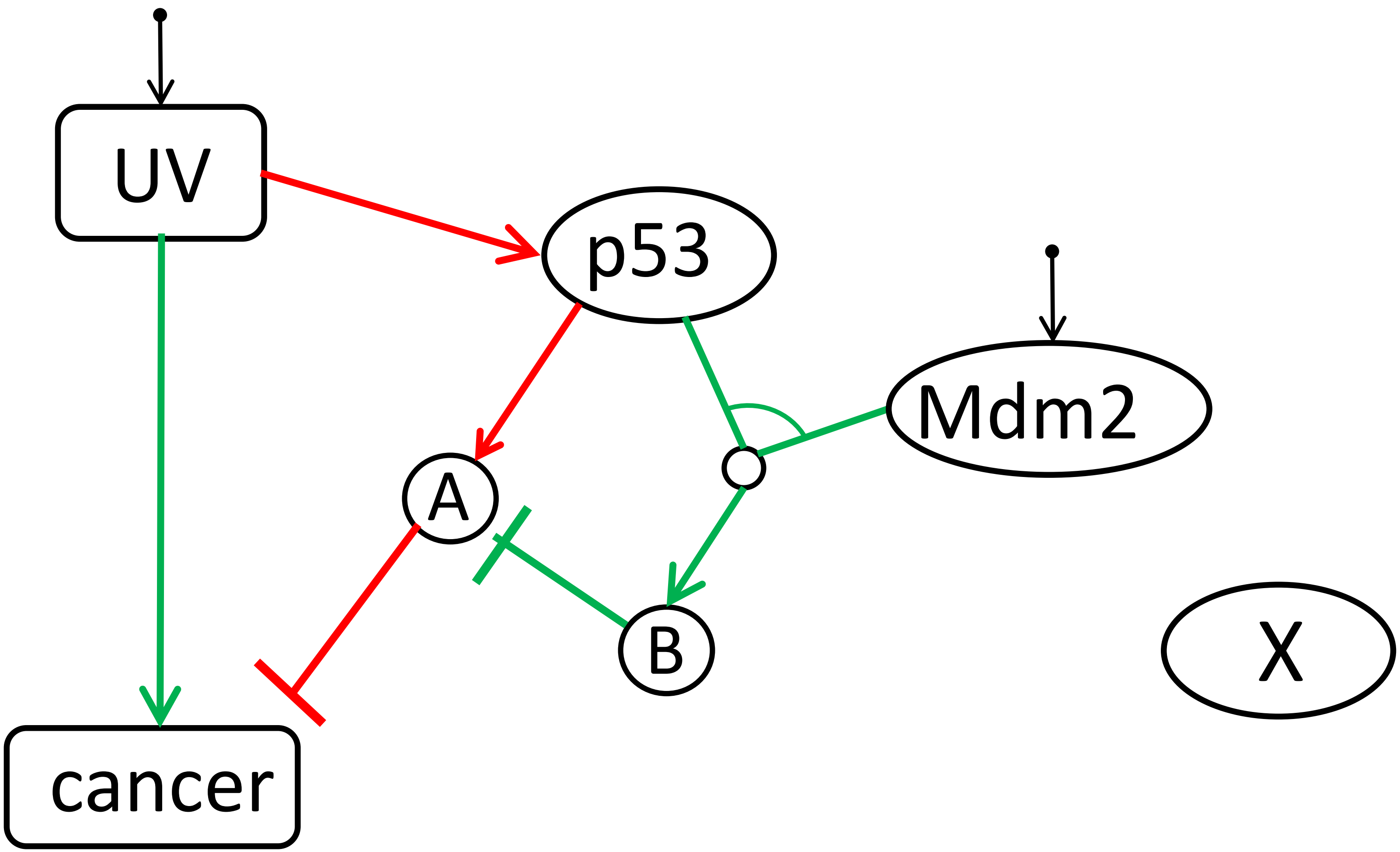
B_0 represents a negative feedback loop.

Theorem (Inoue *et al.*, 2013): B is consistent if and only if there are NO nodes g, s in N such that there are both a proof Π^+ of *promoted*(g, s) and a proof Π^- of *suppressed*(g, s) satisfying (i) neither Π^+ nor Π^- have a trigger; (ii) there are an even number of occurrences of inhibitors in Π^+; and (iii) there are an odd number of occurrences of inhibitors in Π^-.



B_1 : This is an inconsistent network, since both *suppressed*(r, p) and *promoted*(r, p) are derived.

p53 signal network (Tran & Baral, 2009)



Meta-level representation for p53 signal network

triggered(cancer, uv),

triggered(p53, uv),

inhibited(cancer, a),

triggered(a, p53),

inhibited(a, b),

triggered(b, p53 $\wedge$ mdm2),

triggered(X, Y $\wedge$ Z) $\equiv$ (triggered(X, Y) $\vee$ triggered(X,Z)).

Goal and abducibles for p53 signal network

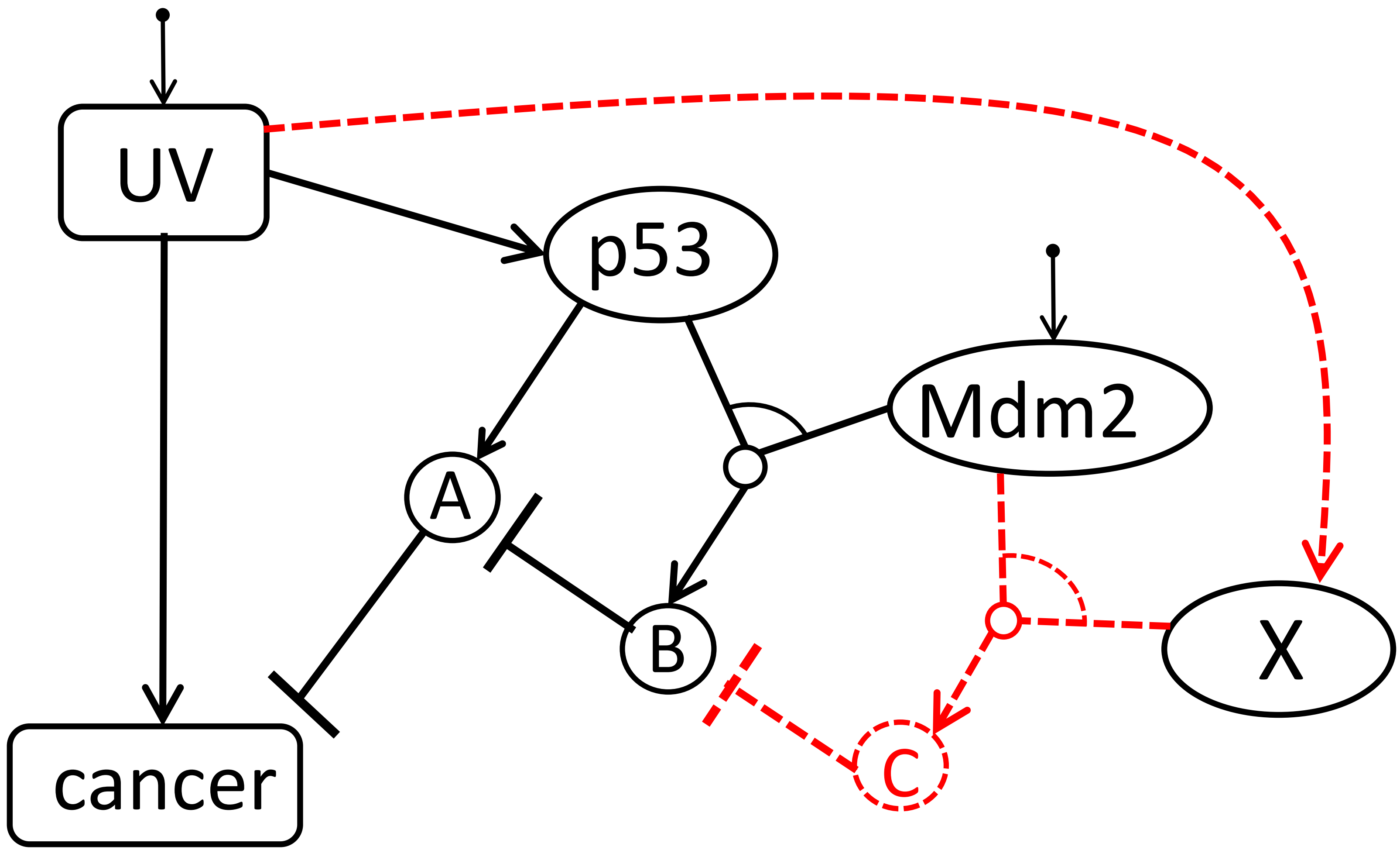
- Consider a tumor suppressor gene X such that mutants of X are highly susceptible to cancer. Suppose exposure of the cell to high level UV does not lead to cancer, given that the initial concentration of Mdm2 is high. These initial conditions are represented as

$$source(uv) \ \wedge \ source(mdm2),$$

i.e., both UV and Mdm2 can be abduced whenever necessary.

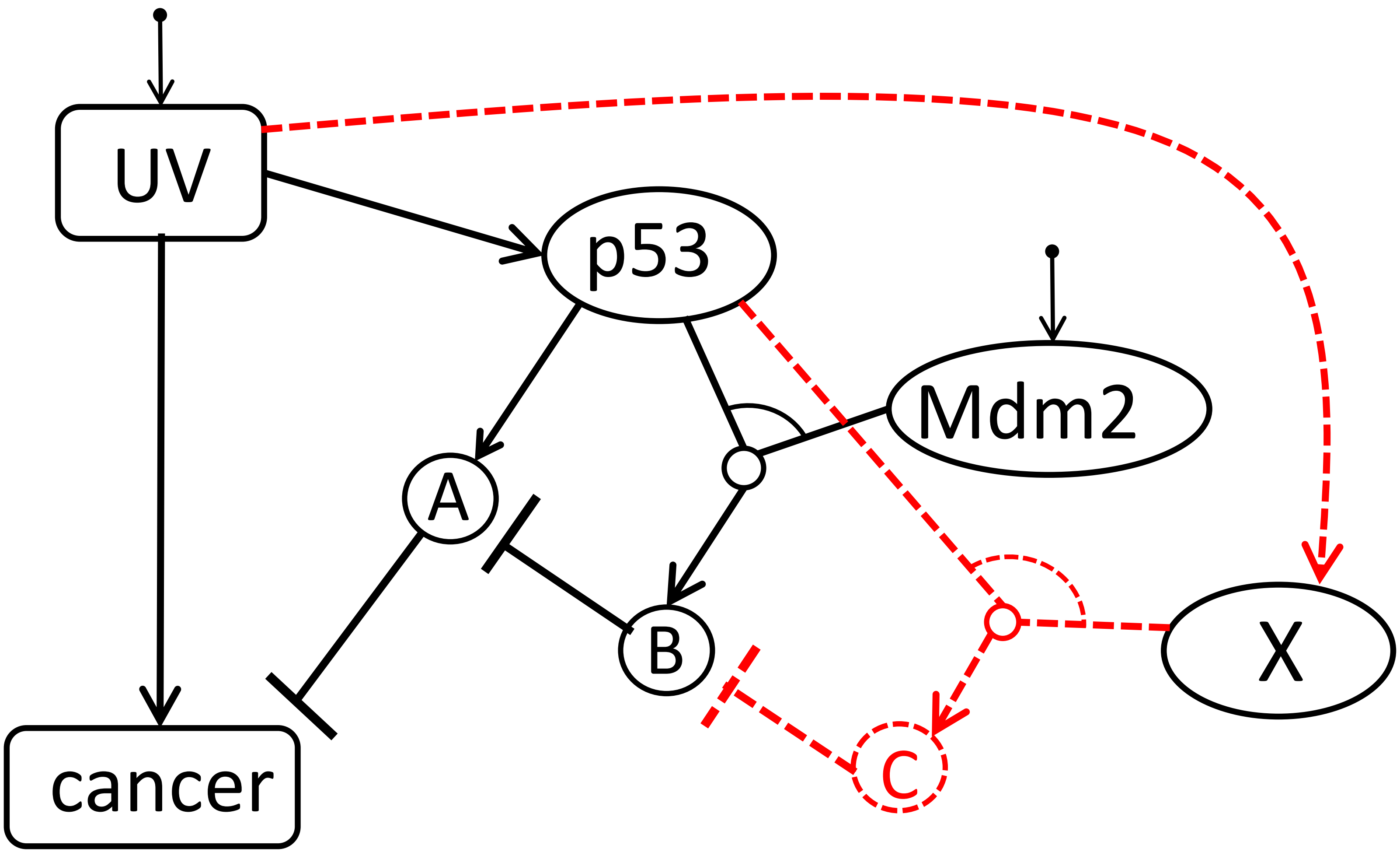
- **Objective**: hypothesize about the possible influences of X on the p53 pathway, explaining how the cell can avoid cancer.
- **Goal**: $\exists S\ (suppressed(cancer, S) \wedge source(S))$
- **Abducibles**: $\Gamma = \{triggered(\_,\_), inhibited(\_,\_)\}$
- **Top clause**: $(\neg suppressed(cancer, S) \vee \neg source(S) \vee ans(S))$
- **Production field**: $\{\neg L \mid L \in \Gamma\} \ \cup \ \{ ans(\_) , \neg no\_inhibitor(\_) \}$
- SOLAR produces 26 minimal hypotheses in 3 seconds.

Hypothesis I



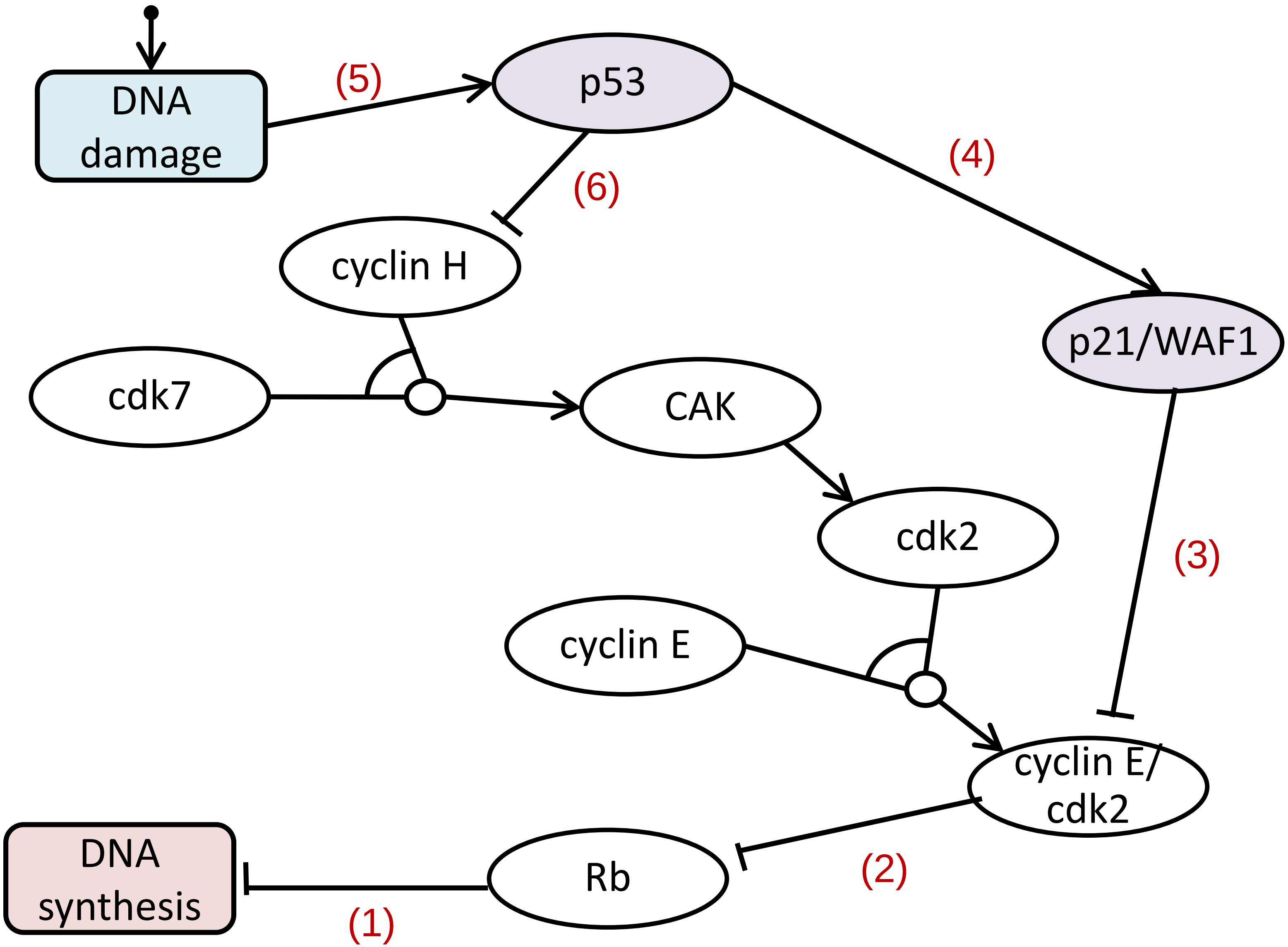
triggered(x, uv) $\wedge$ $\exists$ *Y* (*triggered(Y, mdm2* $\wedge$ *x)* $\wedge$ *inhibited(b, Y)*)

Hypothesis II



triggered(x, uv) $\wedge$ $\exists$ *Y* (*triggered(Y, p53* $\wedge$ *x)* $\wedge$ *inhibited(b, Y)*)

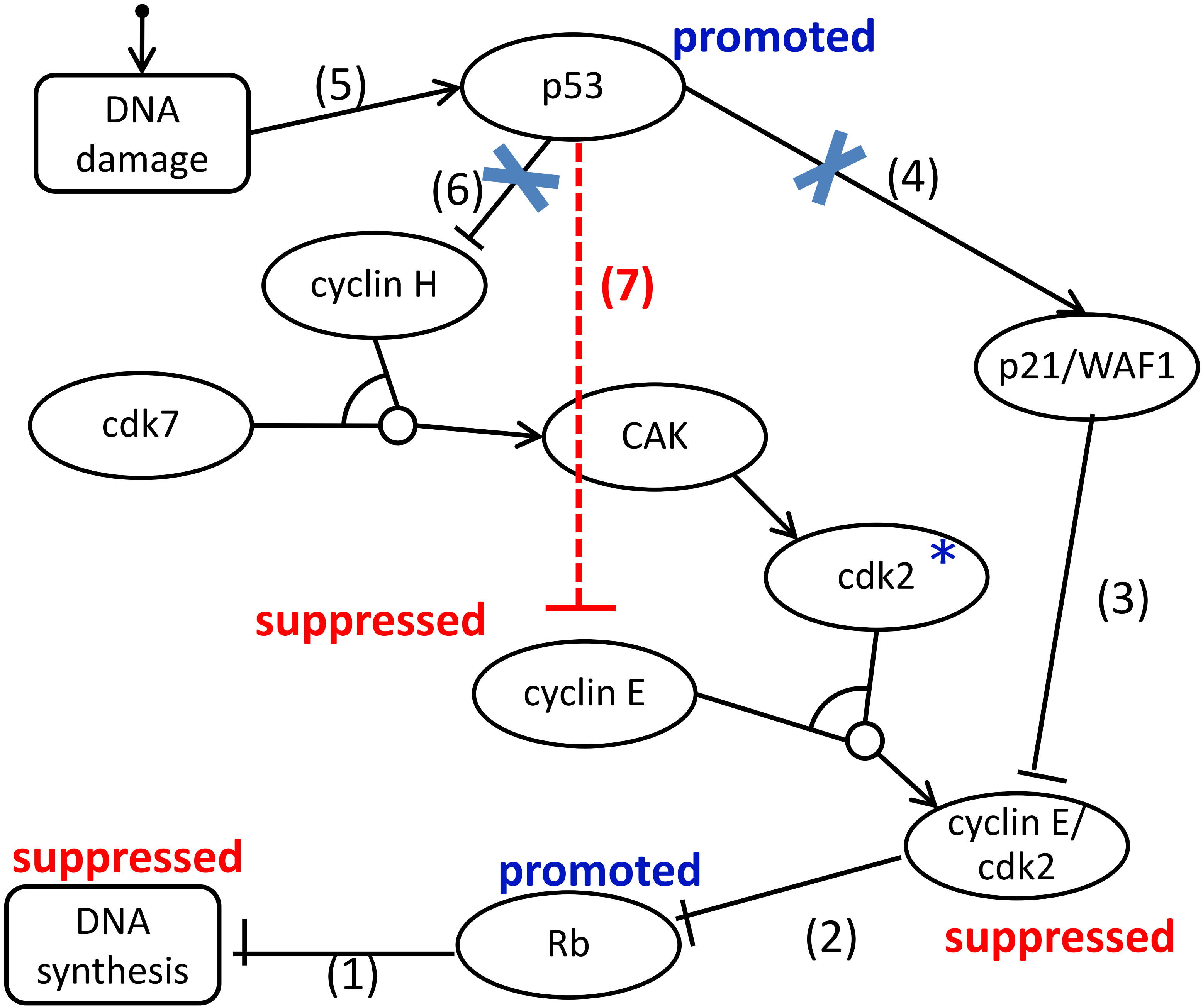
Cyclin-dependent kinases (Schneider *et al.*, 2002)



Cut-off experiment of CDK network

- The original CDK network is cut off at (1)—(6).
- Whether the same links are restored by meta-level abduction?
- Interestingly, the network has a redundancy (→robustness).
- Some links are recovered by being generalized. E.g., {(1g),(2g)} becomes $\exists X(\mathit{inhibited}(\mathit{dna\_synthesis}, X) \wedge \mathit{inhibited}(X, \mathit{cyclin\ e/cdk2}))$
- New pathways are also created. (7) corresponds to **analogy**.

Removed links	#H	P21-mediated	Cdk2 active	cyclin E inhibit
(1)	4	(1)	—	—
(1),(2)	9	(1g),(2g)	—	—
(1),(3)	17	(1),(3)	(1)	(7)
(2),(4)	24	(2),(4)	(2)	(7)
(4),(6)	22	(4)	(6)	(7)
(1),(2),(3),(4),(5)	392	(1g),(2g),(3g),(4),(5)	(1g),(2g),(5)	(1g),(2g),(5),(7)



promoted
DNA damage
(5)
p53
(6)
(4)
(7)
cyclin H
cdk7
CAK
p21/WAF1
cdk2 *
(3)
suppressed
cyclin E
cyclin E/ cdk2
suppressed
promoted
DNA synthesis
Rb
(1)
(2)
suppressed

Law Meta-Level Abduction

Kind of abduction	Explanandum	Explanans
Factual abduction	Single facts	New facts
— Observable-fact abduction	Single facts	Factual reasons
— 1st-order existential abduction	Single facts	Facts with new unknown individuals
— Unobservable-fact abduction	Single facts	Unobservable facts
Law abduction	Empirical laws	New laws
Theoretical-model abduction	General empirical phenomena	New theoretical models
2nd-order existential abduction	General empirical phenomena	New laws with new concepts
— Micro-part abduction	General empirical phenomena	Microscopic compositions
— Analogical abduction	General empirical phenomena	New laws with analogical concepts
— Hypothetical cause abduction	General empirical phenomena	Hidden (unobservable) causes

Meta-level

Law meta-level abduction

- Could be called “meta-level induction”
- Theoretically, any ILP system that is capable of law abduction can be used for law MLA.
- By law MLA, we can devise a part of deductive or abductive system.
- For example, let T be

  $(caused(X,Y) \leftarrow linked(X,Z) \wedge caused(Z,Y)) \wedge linked(a, b) \wedge linked(b, c)$, and G = $caused(a, c)$. Then, CF-induction (Inoue, 2014) can induce a meta-axiom: $(caused(X,Y) \leftarrow linked(X,Y))$. This is abduction of a meta-axiom for SLC.

Representational Meta-Level Abduction

Kind of abduction	Explanandum	Explanans
Factual abduction	Single facts	New facts
— Observable-fact abduction	Single facts	Factual reasons
— 1st-order existential abduction	Single facts	…w unknown
— Unobservable-fact abduction	Single facts	Unobservable facts
Law abduction	Empirical laws	New laws
Theoretical-model abduction	General empirical phenomena	New theoretical models
2nd-order existential abduction	General empirical phenomena	New laws with new concepts
— Micro-part abduction	General empirical phenomena	Microscopic compositions
— Analogical abduction	General empirical phenomena	New laws with analogical concepts
— Hypothetical cause abduction	General empirical phenomena	Hidden (unobservable) causes

Meta-level

Representational meta-level abduction (thought experiment)

- A representation change often appears in scientific discovery as a paradigm shift.
- For example, non-Euclidean geometry changed geometrical postulates of Euclidean geometry. In the course of such a discovery, a new axiom set is introduced into the new framework. Hence abducing an essential change in a domain is representation abduction at the object level.
- For such invented non-Euclidean geometries, Hilbert studied axiomatic systems of both Euclidian and non-Euclidian geometries in his *"Foundations of Geometry"*. As Hilbert later invoked the term *metamathematics*, his work on axiomatic systems exactly describes a meta-theory of those variants of geometries. This is exactly a case of representation MLA.

Can machines learn logics?

- Extraction of a hidden logic from a domain or inventing a new logic from observations in a new situation invokes representation MLA.
- Sakama and Inoue (2015) consider the question:
  - There are two machines *A* and *B*. The machine *A* is capable of deductive reasoning with an underlying logic *L*. Given a set *S* of formulas as an input, the machine *A* produces (a subset of) the logical consequences *Th*(*S*) as an output. The machine *B* has no axiomatic system for deduction. Given input-output pairs (*S*1*, Th*(*S*1))*, . . . ,* (*Si, Th*(*Si*))*, . . .* of *A* as an input to *B*, the problem is whether one can develop an algorithm *C* which successfully produces the axiomatic system *L* for deduction.
- Meta-level one-step deduction rules including MP are learned.
- The scenario can be applied to learning abduction and other non-standard logics.

Problem solving with meta-level abduction

- Consists of:
  1. **design of meta-level axioms**,
  2. **representation of domain knowledge at the meta level,**
  3. **restriction of the search space to treat large knowledge.**
- The task (2) is rather tractable.
- Law abduction can contribute to the task (1) is important. But other axiomatizations are considerable, e.g., introduction of time, modality, majority logic.
- The task (3) can be realized by introducing more constraints. Automation of constraint generation is future work.
- Hypothesis evaluation/ranking is also important, c.f. (Inoue *et al*., IJCAI-09), (Gat-Viks & Shamir, 2002).
- Analogy is useful to get a particular form of hypotheses.

Conclusion/Perspectives

- The method of **meta-level abduction** has been analyzed from the viewpoint of patterns of abduction.
- Fact MLA is strong enough to realize 2nd-order existential abduction at the meta-level. Two logics of causality, SLC and GLC, have been focused in the form of causal networks.
- *New nodes* can also be abduced and ***predicate invention*** is realized as *existentially quantified hypotheses*.
- *Law abduction* is another form of ***induction***—SOLAR as an inductive inference engine.
- Meta-level abduction has been extended to allow *abduction of positive and negative links* and *abduction with defaults*.
- Law MLA can produce meta-axioms in logics. Representation MLA can abduce logics themselves.