Meta-Level Abduction

Katsumi Inoue
National Institute of Informatics, Japan

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Meta-level abduction —Contents—

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Meta-reasoning

• Pereira, Bundy, Russel, et al.
  – The meta-level control of the reasoning process itself.
  – The introspective monitoring of the reasoning process at the object level.

• Kowalski initiated *meta-programming* in logic programming, e.g., [Bowen & Kowalski, 1982]
“Vanilla” meta-interpreter (Hill & Lloyd)

\[
solve(\text{true}).
\]
\[
solve(\text{A} \land \text{B}) \leftarrow solve(\text{A}) \land solve(\text{B}).
\]
\[
solve(\neg \text{A}) \leftarrow \neg solve(\text{A}).
\]
\[
solve(\text{A}) \leftarrow rule(\text{A} \leftarrow \text{B}) \land solve(\text{B}).
\]

- In general, any inference rule can be expressed in such a meta-rule, e.g.,
  \[
solve(\text{A} \leftarrow \text{B}) \leftarrow solve(\text{A} \leftarrow \text{C}) \land solve(\text{C} \leftarrow \text{B}).
  \]
  \[
solve(\neg \text{A}) \leftarrow solve(\text{B} \leftarrow \text{A}) \land solve(\neg \text{B}).
  \]

- All constructs with predicates `solve`, `clause`, `demo` are atoms.
- Those meta-level axioms are used for *deduction* only.
Abductive inference

• Abduction augments *sufficient conditions* missing in the premises (background knowledge) to enable a derivation (proof) of the given observation (goal).

• This inference *fills the gap* in a proof of the observation from the premises.

• Inferred sufficient conditions are called *hypotheses* or *explanations*. Often, they are called *explanans*. The observation is called the *explanundum*.

• A hypothesis can be any formula, e.g., a set (conjunction) of atoms/literals/rules.
Three modes of inference
(C.S. Peirce)

- **Deduction**
  \[ A \to C \]
  \[ \frac{A}{C} \]

- **Induction**
  \[ A \rightarrow C \]
  \[ \frac{C}{A \rightarrow C} \]

- **Abduction**
  \[ A \to C \]
  \[ \frac{C}{A} \]
Meta-level abduction

• *Abduction is performed on meta-level axioms.*
• For example, given a meta-theory:
  \[ \text{solve}(A) \leftarrow \text{rule}(A \leftarrow B) \land \text{solve}(B), \]
  \[ \text{solve}(B), \]
  and a meta-goal:
  \[ \text{solve}(A), \]
  we can abduce
  \[ \text{rule}(A \leftarrow B). \]
• In this example, we can realize *rule abduction.*
• Yet, this is an ordinary abduction since it abduce atoms.
Meta-level abduction (Inoue et al., 2009-)

- A method to abduce rules, enabling to infer hypotheses
  - explaining empirical rules by means of hidden rules,
  - representing multiple missing links / causal relations,
  - simultaneously accounting for multiple observations,
  - containing unknown nodes as new predicates.
  - A new way of induction based on full-clausal abduction.

- Combination of rule abduction and fact abduction is possible by way of conditional query answering.

- First motivating examples were from cognitive modeling for improving the skill of music playing (ILP ‘09, LNAI 5989)

- The method has been applied to scientific discovery from network data, e.g., genetic/biochemical pathways (ILP ‘10, LNAI 6489; Machine Learning 91, 2013; FMMB ‘14, LNBI 8738).
## Patterns of Abduction (Schurz, *Synthese*, 2008)

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## Factual Meta-Level Abduction

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A simple logic of causality

- To express relations between events, we use causal chains.
- Causality can be represented in first-order predicate logic.
- Two meta-predicates:
  1. $\text{linked}(X,Y)$: $X$ is directly caused by $Y$.
  2. $\text{caused}(X,Y)$: There is a causal chain from $Y$ to $X$.

- Basic axioms:

  \[
  \text{caused}(X,Y) \leftarrow \text{linked}(X,Y).
  \]
  \[
  \text{caused}(X,Y) \leftarrow \text{linked}(X,Z) \land \text{caused}(Z,Y).
  \]
Representing logical connectives

\[\text{linked}(g, s).\]

\[\neg \text{linked}(g, s).\]

\[\text{linked}(g, s) \lor \text{linked}(h, s)\]

\[\text{linked}(g, s) \lor \text{linked}(g, t)\]
Object and meta level representation

- **Object domain (object level)**
  
  \[ A \leftarrow B. \]
  
  \[ B \leftarrow C \land D. \]

- Each *rule* in the object level is represented as a *fact* in the meta level.

- Each *literal* in the object level is represented as a *term* in the meta level.

- **Causal relations (meta level)**
  
  \[ \text{linked}(A, B). \]
  
  \[ \text{linked}(B, C) \lor \text{linked}(B, D). \]

- *Rule abduction* in the object level is realized by abducing literals of the form \( \text{linked}(_, _) \) at the meta level.
Formalizing rule abduction

- $g$: a goal, $s$: an input, $r$: a (hidden) node

**B:** $\text{linked}(g, r)$.

$\neg \text{linked}(g, s)$.

That is, $g$ is directly caused by $r$, but $g$ is **not** directly caused by $s$.

- $g$ is not directly caused by $s$, but we know that there is a causal chain to $g$ from $s$.

  This is given by an **observation**:

  **G:** $\text{caused}(g, s)$.

- SOLAR computes a hypothesis

  **H:** $\text{linked}(r, s)$,

  given the abducibles $\{\text{linked}(\_, \_}\}$.
## Patterns of Abduction (Schurz, *Synthese*, 2008)

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Node introduction = Predicate invention

- $g, h$ : goal nodes, $s$ : an input node.

$B$: $\neg \text{linked}(g, s)$.
$\neg \text{linked}(h, s)$.
That is, there are no direct causal relations from $s$ to $g$ and from $h$ to $s$, but there are causal chains as the observations:

$G$: $\text{caused}(g, s) \land \text{caused}(h, s)$.

- Given the abducibles $\{\text{linked}(\_, \_)\}$, SOLAR generates a hypothesis $H$:

  $\exists X. ( \text{linked}(g, X) \land \text{linked}(h, X) \land \text{linked}(X, s) )$.

- Variable $X$ represents a newly introduced node, which corresponds to predicate invention or object invention.
Representing different structures

\( B: \neg \text{linked}(g, s). \)
\( \neg \text{linked}(h, s). \)

\( G: \text{caused}(g, s). \)
\( \text{caused}(h, s). \)

Abducibles: \{\text{linked}(\_, \_)}\}.

H with 2 intermediate nodes:

\( \exists X \exists Y. ( \text{linked}(g, X) \land \text{linked}(h, Y) \land \text{linked}(X, s) \land \text{linked}(Y, s) ). \)

\( \exists X \exists Y. ( \text{linked}(g, X) \land \text{linked}(h, Y) \land \text{linked}(X, Y) \land \text{linked}(Y, s) ). \)
Correctness of meta-level abduction

**Proposition:** Let $\lambda(B)$ be the theory obtained by replacing every $\text{linked}(g, s)$ appearing in $B$ with the formula $(g \leftarrow s)$.

$B \cup \text{Meta-Axioms} \models (\text{caused}(g, s_1) \lor \cdots \lor \text{caused}(g, s_n))$

iff $\lambda(B) \models (g \leftarrow s_1 \land \cdots \land s_n)$.

**Theorem:** Observation: $O = (\text{caused}(g, s_1) \lor \cdots \lor \text{caused}(g, s_n))$.

$H$ is an abductive explanation of $O$ with respect to $B$ and $\Gamma_M = \{\text{linked}(_,_)_\}$ iff $\lambda(H)$ is a hypothesis such that

- $\lambda(B) \cup \lambda(H) \models (g \leftarrow s_1 \land \cdots \land s_n)$, and
- $\lambda(B) \cup \lambda(H)$ is consistent.
Implementation

• Although the idea of meta-level abduction is simple, its implementation requires an abductive procedure for first-order full clausal theories.

• Currently, SOLAR (Nabeshima et al., 2010) is only such a state-of-the-art procedure.

• There will be a possibility to use logic programming and answer set programming.
Application to “knack” discovery

B: linked(inc_sound, bow_close_to_the_bridge).
   linked(bow_close_to_the_bridge, stable_bow_movement) ∨
   linked(bow_close_to_the_bridge, smooth_bow_direction_change).
   linked(smooth_bow_direction_change, flexible_wrist).
   ← linked(inc_sound, keep_arm_close).
   ← linked(stable_bow_movement, keep_arm_close).
   ← linked(smooth_bow_direction_change, keep_arm_close).

G: caused(inc_sound, keep_arm_close).

● SOLAR generates 52 hypotheses (maximum search depth: 15, maximum length of produced clauses: 5). One of them is:

∃ X. (linked(stable_bow_movement, X) ∧ linked(flexible_wrist, X) ∧ linked(X, keep_arm_close)).
The obtained hypothesis

\( H: \exists X. (\text{linked}(\text{stable}_\text{bow}_\text{movement}, X) \land \text{linked}(\text{flexible}_\text{wrist}, X) \land \text{linked}(X, \text{keep}_\text{arm}_\text{close}) ). \)
Abducting facts

New axioms:

\[
\text{caused}(X, X) \leftarrow \text{abd}(X). \quad \% \text{ for abducibles}
\]

\[
\text{caused}(X, Y) \leftarrow \text{linked}(X, Y).
\]

\[
\text{caused}(X, Y) \leftarrow \text{linked}(X, Z) \land \text{caused}(Z, Y).
\]

The top clause:

\[
\leftarrow \text{caused}(g, X) \land \text{abd}(X).
\]

Note: \text{abd} plays the role of an \textit{answer predicate}.

An integrity constraint that \(p\) and \(q\) cannot hold simultaneously:

\[
\leftarrow \text{caused}(p, X) \land \text{caused}(q, Y) \land \text{abd}(X) \land \text{abd}(Y).
\]
Abducing facts and rules

• Abducing facts is nothing but answer extraction.

• Abducing facts and rules is then conditional query answering.
## Patterns of Abduction (Schurz, *Synthese*, 2008)

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General Logic of Causality (GLC)

• Now we do not consider any particular interpretation of the meta-predicate *linked* so that its semantics is left open.
• This is desirable, since *caused*(\(g, s\)) cannot be interpreted as the material implication \((g \leftarrow s)\) in general. Instead, the meaning of causality at the object level is now abstracted away.
• In particular, the correspondence such that \((\text{caused}(g, s_1) \lor \cdots \lor \text{caused}(g, s_n)) \iff (g \leftarrow s_1 \land \cdots \land s_n)\) is abandoned.
• In GLC, a causal network is given as a set of atoms of the form *linked*(\(F,G\)), where \(F\) and \(G\) are any Boolean formulas. Similarly, atoms of the form *caused*(\(F,G\)) are allowed with any \(F\) and \(G\).
• This logic, called GLC, is defined by *structural inference rules*.
• Here, we denote the meta-atoms *linked*(\(g, s\)) and *caused*(\(g, s\)) as the meta-relations \((s \triangleright g)\) and \((s \models g)\), respectively.
Postulates for GLC

• (Reflection) \[ \frac{\alpha \triangleright \psi}{\alpha \vdash \psi} \]

• (Weakening) \[ \frac{\alpha \vdash \theta \quad \theta \gg \psi}{\alpha \vdash \psi} \]

• (Strengthening) \[ \frac{\alpha \gg \theta \quad \theta \vdash \psi}{\alpha \vdash \psi} \]

• (And) \[ \frac{\alpha \vdash \varphi \quad \alpha \vdash \psi}{\alpha \vdash \varphi \land \psi} \]

• (Cut) \[ \frac{\alpha \vdash \theta \quad \alpha \land \theta \vdash \psi}{\alpha \vdash \psi} \]

• (Or) \[ \frac{\alpha \vdash \psi \quad \theta \vdash \psi}{\alpha \lor \theta \vdash \psi} \]

Note: \( \gg \) is a classical Tarski consequence operator; \( X \gg Y \) iff \( X \) entails \( Y \).
GLC meta-program

\[
\text{caused}(X, Y) \leftarrow \text{linked}(X, Y).
\]

\[
\begin{align*}
\text{caused}(X, Y) & \leftarrow (X \leq Z) \land \text{caused}(Z, Y). \\
\text{caused}(X, Y) & \leftarrow \text{caused}(X, Z) \land (Z \leq Y). \\
\text{caused}(X \land Y, Z) & \leftarrow \text{caused}(X, Z) \land \text{caused}(Y, Z). \\
\text{caused}(X, Y) & \leftarrow \text{caused}(X, Y \land Z) \land \text{caused}(Z, Y). \\
\text{caused}(X, Y \lor Z) & \leftarrow \text{caused}(X, Y) \land \text{caused}(X, Z).
\end{align*}
\]

Note: \( \leq \) is a classical consequence operator;

\[
X \leq Y \text{ iff } Y \geq X \text{ iff } Y \text{ entails } X, \text{ e.g., } s \leq s \land t
\]
GLC is more general than SLC

**Proposition.** Suppose a theory $T$ for SLC, and let $T'$ be a theory for GLC obtained from $T$ by replacing every disjunctive fact of the form $(\text{linked}(p, q_1) \lor \cdots \lor \text{linked}(p, q_m))$ with a fact of the form $\text{linked}(p, q_1 \land \cdots \land q_m)$. If $G = (\text{caused}(g, s_1) \lor \cdots \lor \text{caused}(g, s_n))$ is derived from $T$ in SLC, then $G' = \text{caused}(g, s_1 \land \cdots \land s_n)$ is derived from $T'$ in GLC.

**Proof (sketch):** Reflection in GLC corresponds to the first meta-axiom in SLC. The second meta-axiom in SLC is obtained by Transitivity (which is a consequence of Strengthening and Cut) and Reflection.

The case inference in SLC can be simulated in GLC as follows. Suppose $(\text{linked}(q, p_1) \lor \text{linked}(q, p_2))$ in SLC. From $\text{caused}(p_1, s_1)$ and $\text{caused}(p_2, s_2)$, $(\text{caused}(q, s_1) \lor \text{caused}(q, s_2))$ is derived in SLC. The corresponding inference can be obtained in GLC as:

\[
\begin{align*}
  s_1 & \not\models p_1 & s_2 & \not\models p_2 \\
  s_1 \land s_2 & \not\models p_1 & s_1 \land s_2 & \not\models p_2 \\
  s_1 \land s_2 & \not\models p_1 \land p_2 & p_1 \land p_2 & \models q \\
  & s_1 \land s_2 \models q
\end{align*}
\]
Transitivity in GLC

• Strengthening and Cut imply the transitivity:

\[ \alpha \models \beta \quad \beta \models \psi \quad \therefore \quad \alpha \models \psi \]


• But, we can interpret the causality here simply as an *influence*. Moreover, many anomaly cases can be avoided if we carefully model the causal theory.
Monotonicity in GLC

• (Left) Strengthening is also called Monotonicity (Kraus, Lehmann & Magidor, 1990):

\[(\text{Strengthening}) \quad \frac{\alpha \succeq \theta \quad \theta \models \psi}{\alpha \models \psi}\]

• Monotonicity apparently fails to hold in nonmonotonic logics.

• Example. "Striking a match causes it to light" obviously does not imply "Putting a match in water and then striking it causes it to light." (Bochman & Gabbay, 2012)
Networks with positive and negative causal links (Inoue, Doncescu & Nabeshima, 2010-2013)

- Consider networks with both positive and negative causal effects.
- In biology, such networks appear in gene regulatory/transcription systems, signaling networks, and metabolic pathways.
- Two types of direct causal relations: triggered and inhibited.
- **triggered**\( (g, t) \): a positive direct cause (\( t \) is a trigger of \( g \))

  \[
  \begin{array}{c}
  g \\
  \hline
  t 
  \end{array}
  \]

  in a causal graph, whose meaning is \( (g \leftarrow t) \) in the object level, where \( \leftarrow \) means that the causation appears if it is not prevented.

- **inhibited**\( (g, s) \): a negative direct cause (\( s \) is an inhibitor of \( g \))

  \[
  \begin{array}{c}
  g \\
  \hline
  s 
  \end{array}
  \]

  in a causal graph, whose meaning is \( (\neg g \leftarrow s) \) in the object level.
Alternating axioms for causality

- **Causal chains** have two kinds too:
  1. **promoted(X,Y):** X is *positively caused* by Y.
  2. **suppressed(X,Y):** X is *negatively caused* by Y.

\[
\begin{align*}
\text{caused}(X,Y) & \leftarrow \text{linked}(X,Y). \\
\text{caused}(X,Y) & \leftarrow \text{linked}(X,Z) \land \text{caused}(Z,Y).
\end{align*}
\]

\[
\begin{align*}
\text{promoted}(X, Y) & \leftarrow \text{triggered}(X, Y). \\
\text{promoted}(X, Y) & \leftarrow \text{triggered}(X,Z) \land \text{promoted}(Z, Y). \\
\text{promoted}(X, Y) & \leftarrow \text{inhibited}(X,Z) \land \text{suppressed}(Z, Y).
\end{align*}
\]

\[
\begin{align*}
\text{suppressed}(X, Y) & \leftarrow \text{inhibited}(X, Y). \\
\text{suppressed}(X, Y) & \leftarrow \text{inhibited}(X,Z) \land \text{promoted}(Z, Y). \\
\text{suppressed}(X, Y) & \leftarrow \text{triggered}(X,Z) \land \text{suppressed}(Z, Y). \\
\text{promoted}(X, Y) \land \text{suppressed}(X,Y).
\end{align*}
\]
Monotonic property

- Meta-level abduction is defined for an observation $promoted(g, s)$ or $suppressed(g, s)$ with the abducibles
  
  \[ \Gamma = \{ triggered(_,_), \text{ inhibited}(\_,\_) \} . \]

- Given positive and negative observations, both positive and negative direct causes are abduced and new nodes are produced when necessary.

- **Proposition**: For any suppression (resp. promotion) for $g$ from $s$, there is a causal chain $P$ from $s$ to $g$ such that there exist an odd number of (resp. an even number of) direct inhibitions in $P$. 
Antagonistic factors

Suppose that both the trigger $t$ and the suppressor $s$ are activated. Is $g$ promoted or suppressed? Intuitions are:

(1) If $t$ works and $s$ does not work, then $g$ is promoted by $t$;
(2) If $s$ works and $t$ does not work, then $g$ is suppressed by $s$;
(3) If both $t$ and $s$ work, then $g$ is suppressed by $s$.

Namely, an inhibitor is preferred to a trigger.
Axioms with defaults

Causal chains should have *nonmonotonic* effects.

\[
\text{promoted}(X, Y) \leftarrow \text{triggered}(X, Y) \land \text{no_inhibitor}(X).
\]
\[
\text{promoted}(X, Y) \leftarrow \text{triggered}(X,Z) \land \text{no_inhibitor}(X) \land \text{promoted}(Z, Y).
\]
\[
\text{promoted}(X, Y) \leftarrow \text{inhibited}(X,Z) \land \text{suppressed}(Z, Y).
\]
\[
\text{suppressed}(X, Y) \leftarrow \text{inhibited}(X, Y).
\]
\[
\text{suppressed}(X, Y) \leftarrow \text{inhibited}(X,Z) \land \text{promoted}(Z, Y).
\]
\[
\text{suppressed}(X, Y) \leftarrow \text{triggered}(X,Z) \land \text{no_inhibitor}(X) \land \text{suppressed}(Z, Y).
\]
\[
\leftarrow \text{promoted}(X, Y) \land \text{suppressed}(X,Y).
\]

*no_inhibitor(_)* : treated as a *default*, which can be assumed during inference unless contradiction occurs.
Abduction with defaults

• For default assumptions of the form \textit{no\_inhibitor(\_)}, \textit{their negations} are skipped in SOLAR by putting them in the production field.

• Membership of a clause $C$ in an extension of a default theory is guaranteed for each obtained consequence of the form

$$C \leftarrow \textit{no\_inhibitor}(t1) \land \textit{no\_inhibitor}(t2) \land \cdots$$

[Inoue \textit{et al.}, 2004, 2006].
Correspondence between object-level inference and meta-level consequence finding

<table>
<thead>
<tr>
<th>object-level inference</th>
<th>top clause in SOLAR *</th>
<th>production field</th>
</tr>
</thead>
<tbody>
<tr>
<td>proving rules</td>
<td>$\neg \text{caused}(g, s)$</td>
<td>none</td>
</tr>
<tr>
<td>abducing facts</td>
<td>$\neg \text{caused}(g, X) \lor \text{ans}(X)$</td>
<td>$\text{ans}(_)$</td>
</tr>
<tr>
<td>predicting facts</td>
<td>$\neg \text{caused}(X, s) \lor \text{ans}(X)$</td>
<td>$\text{ans}(_)$</td>
</tr>
<tr>
<td>predicting rules</td>
<td>none</td>
<td>$\text{promoted}(_, _), \text{suppressed}(_, _)$</td>
</tr>
<tr>
<td>abducing rules</td>
<td>$\neg \text{caused}(g, s)$</td>
<td>$\neg \text{triggered}(_, _), \neg \text{inhibited}(_, _)$</td>
</tr>
<tr>
<td>abducing rules and facts</td>
<td>$\neg \text{caused}(g, X) \lor \neg \text{abd}(X)$</td>
<td>$\neg \text{triggered}(_, _), \neg \text{inhibited}(_, _), \text{ans}(_)$</td>
</tr>
<tr>
<td>predicting conditional facts</td>
<td>$\neg \text{caused}(X, s) \lor \text{ans}(X)$</td>
<td>$\neg \text{triggered}(_, _), \neg \text{inhibited}(_, _), \text{ans}(_)$</td>
</tr>
<tr>
<td>predicting conditional rules</td>
<td>none</td>
<td>$\neg \text{triggered}(_, _), \neg \text{inhibited}(_, _), \text{promoted}(_, _), \text{suppressed}(_, _)$</td>
</tr>
</tbody>
</table>

* $\neg \text{caused}(X,Y)$ is instantiated by either $\neg \text{promoted}(X,Y)$ or $\neg \text{suppressed}(X,Y)$. 
Consistency of meta-level abduction

\[ B_0 = \{ \text{inhibited}(p, p) \}. \]

\( B_0 \cup \text{Meta-Axioms} \) is inconsistent.

\( B_0 \) represents a negative feedback loop.

**Theorem (Inoue et al., 2013):** \( B \) is consistent if and only if there are NO nodes \( g, s \) in \( N \) such that there are both a proof \( \Pi^+ \) of \( \text{promoted}(g, s) \) and a proof \( \Pi^- \) of \( \text{suppressed}(g, s) \) satisfying (i) neither \( \Pi^+ \) nor \( \Pi^- \) have a trigger; (ii) there are an even number of occurrences of inhibitors in \( \Pi^+ \); and (iii) there are an odd number of occurrences of inhibitors in \( \Pi^- \).

\( B_1 : \) This is an inconsistent network, since both \( \text{suppressed}(r, p) \) and \( \text{promoted}(r, p) \) are derived.
p53 signal network (Tran & Baral, 2009)

UV
\[\rightarrow\]
cancer

A
\[\rightarrow\]
B

\[\rightarrow\]
Mdm2

X

p53
Meta-level representation for p53 signal network

\[
\text{triggered(cancer, uv),}
\]
\[
\text{triggered(p53, uv),}
\]
\[
\text{inhibited(cancer, a),}
\]
\[
\text{triggered(a, p53),}
\]
\[
\text{inhibited(a, b),}
\]
\[
\text{triggered(b, p53 } \land \text{ mdm2),}
\]
\[
\text{triggered(X, Y } \land \text{ Z) } \equiv (\text{triggered(X, Y ) } \lor \text{ triggered(X,Z)).}
\]
Goal and abducibles for p53 signal network

- Consider a tumor suppressor gene X such that mutants of X are highly susceptible to cancer. Suppose exposure of the cell to high level UV does not lead to cancer, given that the initial concentration of Mdm2 is high. These initial conditions are represented as

\[ \text{source}(uv) \land \text{source}(mdm2), \]

i.e., both UV and Mdm2 can be abduced whenever necessary.

- **Objective:** hypothesize about the possible influences of X on the p53 pathway, explaining how the cell can avoid cancer.

- **Goal:** \( \exists S (\text{suppressed}(cancer, S) \land \text{source}(S)) \)

- **Abducibles:** \( \Gamma = \{\text{triggered}(\_,\_), \text{inhibited}(\_,\_)\} \)

- **Top clause:** \( (\neg \text{suppressed}(cancer, S) \lor \neg \text{source}(S) \lor \text{ans}(S)) \)

- **Production field:** \( \{\neg L \mid L \in \Gamma\} \cup \{\text{ans}(\_), \neg \text{no_inhibitor}(\_)\} \)

- SOLAR produces 26 minimal hypotheses in 3 seconds.
Hypothesis I

\[
\text{triggered}(x, uv) \land \exists Y (\text{triggered}(Y, \text{mdm2} \land x) \land \text{inhibited}(b, Y))
\]
Hypothesis II

\[ \text{triggered}(x, \text{uv}) \land \exists Y (\text{triggered}(Y, p53 \land x) \land \text{inhibited}(b, Y)) \]
Cyclin-dependent kinases (Schneider *et al.*, 2002)

- DNA damage
- cyclin H
- cdk7
- CAK
- cdk2
- cyclin E
- Rb
- p53
- p21/WAF1
- cyclin E/cdk2

1. DNA synthesis
2. Rb
3. cyclin E/cdk2
4. p21/WAF1
5. p53
6. cyclin H
Cut-off experiment of CDK network

- The original CDK network is cut off at (1)—(6).
- Whether the same links are restored by meta-level abduction?
- Interestingly, the network has a redundancy (→robustness).
- Some links are recovered by being generalized. E.g., \{(1g),(2g)\} becomes \(\exists X(inhibited(dna\_synthesis, X) \land inhibited(X,\ cyclin\ e/cdk2))\)
- New pathways are also created. (7) corresponds to analogy.

<table>
<thead>
<tr>
<th>Removed links</th>
<th>#H</th>
<th>P21-mediated</th>
<th>Cdk2 active</th>
<th>cyclin E inhibit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>4</td>
<td>(1)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(1),(2)</td>
<td>9</td>
<td>(1g),(2g)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(1),(3)</td>
<td>17</td>
<td>(1),(3)</td>
<td>(1)</td>
<td>(7)</td>
</tr>
<tr>
<td>(2),(4)</td>
<td>24</td>
<td>(2),(4)</td>
<td>(2)</td>
<td>(7)</td>
</tr>
<tr>
<td>(4),(6)</td>
<td>22</td>
<td>(4)</td>
<td>(6)</td>
<td>(7)</td>
</tr>
<tr>
<td>(1),(2),(3),(4),(5)</td>
<td>392</td>
<td>(1g),(2g),(3g),(4),(5)</td>
<td>(1g),(2g),(5)</td>
<td>(1g),(2g),(5),(7)</td>
</tr>
</tbody>
</table>
DNA damage → p53

DNA synthesis

cyclin E → Rb

(1) DNA damage promoted

(2) Rb suppressed

(3) cyclin E/cdk2

(4) p53 suppressed

(5) DNA damage promoted

(6) cyclin H suppressed

(7) cyclin E/cdk2 suppressed

p21/WAF1

cdk2

CAK

suppressed

promoted

suppressed
<table>
<thead>
<tr>
<th>Kind of abduction</th>
<th>Explanandum</th>
<th>Explanans</th>
</tr>
</thead>
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<tr>
<td><strong>Factual abduction</strong></td>
<td>Single facts</td>
<td>New facts</td>
</tr>
<tr>
<td>— Observable-fact abduction</td>
<td>Single facts</td>
<td>Factual reasons</td>
</tr>
<tr>
<td>— 1st-order existential abduction</td>
<td>Single facts</td>
<td>Facts with new unknown individuals</td>
</tr>
<tr>
<td>— Unobservable-fact abduction</td>
<td>Single facts</td>
<td>Unobservable facts</td>
</tr>
<tr>
<td><strong>Law abduction</strong></td>
<td>Empirical laws</td>
<td>New laws</td>
</tr>
<tr>
<td><strong>Theoretical-model abduction</strong></td>
<td>General empirical phenomena</td>
<td>New theoretical models</td>
</tr>
<tr>
<td><strong>2nd-order existential abduction</strong></td>
<td>General empirical phenomena</td>
<td>New laws with new concepts</td>
</tr>
<tr>
<td>— Micro-part abduction</td>
<td>General empirical phenomena</td>
<td>Microscopic compositions</td>
</tr>
<tr>
<td>— Analogical abduction</td>
<td>General empirical phenomena</td>
<td>New laws with analogical concepts</td>
</tr>
<tr>
<td>— Hypothetical cause abduction</td>
<td>General empirical phenomena</td>
<td>Hidden (unobservable) causes</td>
</tr>
</tbody>
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Law meta-level abduction

• Could be called “meta-level induction”
• Theoretically, any ILP system that is capable of law abduction can be used for law MLA.
• By law MLA, we can devise a part of deductive or abductive system.
• For example, let T be

\[
(caused(X,Y) \leftarrow linked(X,Z) \land caused(Z,Y)) \land
linked(a, b) \land linked(b, c), \text{ and } G = caused(a, c).
\]

Then, CF-induction (Inoue, 2014) can induce a meta-axiom: \((caused(X,Y) \leftarrow linked(X,Y))\). This is abduction of a meta-axiom for SLC.
# Representational Meta-Level Abduction

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Representational meta-level abduction (thought experiment)

• A representation change often appears in scientific discovery as a paradigm shift.

• For example, non-Euclidean geometry changed geometrical postulates of Euclidean geometry. In the course of such a discovery, a new axiom set is introduced into the new framework. Hence abducting an essential change in a domain is representation abduction at the object level.

• For such invented non-Euclidean geometries, Hilbert studied axiomatic systems of both Euclidian and non-Euclidian geometries in his “Foundations of Geometry”. As Hilbert later invoked the term *metamathematics*, his work on axiomatic systems exactly describes a meta-theory of those variants of geometries. This is exactly a case of representation MLA.
Can machines learn logics?

• Extraction of a hidden logic from a domain or inventing a new logic from observations in a new situation invokes representation MLA.

• Sakama and Inoue (2015) consider the question:
  – There are two machines A and B. The machine A is capable of deductive reasoning with an underlying logic L. Given a set S of formulas as an input, the machine A produces (a subset of) the logical consequences \( Th(S) \) as an output. The machine B has no axiomatic system for deduction. Given input-output pairs \( (S_1, Th(S_1)), \ldots, (S_i, Th(S_i)), \ldots \) of A as an input to B, the problem is whether one can develop an algorithm C which successfully produces the axiomatic system L for deduction.

• Meta-level one-step deduction rules including MP are learned.

• The scenario can be applied to learning abduction and other non-standard logics.
Problem solving with meta-level abduction

- Consists of:
  1. design of meta-level axioms,
  2. representation of domain knowledge at the meta level,
  3. restriction of the search space to treat large knowledge.
- The task (2) is rather tractable.
- Law abduction can contribute to the task (1) is important. But other axiomatizations are considerable, e.g., introduction of time, modality, majority logic.
- The task (3) can be realized by introducing more constraints. Automation of constraint generation is future work.
- Hypothesis evaluation/ranking is also important, c.f. (Inoue et al., IJCAI-09), (Gat-Viks & Shamir, 2002).
- Analogy is useful to get a particular form of hypotheses.
Conclusion/Perspectives

• The method of **meta-level abduction** has been analyzed from the viewpoint of patterns of abduction.

• Fact MLA is strong enough to realize 2nd-order existential abduction at the meta-level. Two logics of causality, SLC and GLC, have been focused in the form of causal networks.

• *New nodes* can also be abduced and **predicate invention** is realized as *existentially quantified hypotheses*.

• *Law abduction* is another form of **induction**—SOLAR as an inductive inference engine.

• Meta-level abduction has been extended to allow **abduction of positive and negative links** and **abduction with defaults**.

• Law MLA can produce meta-axioms in logics. Representation MLA can abduce logics themselves.