

(NON)-MAXIMALITY AND DISTRIBUTIVITY: A DECISION THEORY APPROACH*

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Vagueness in questions. van Rooy (2003) uses Decision Theory (DT) to give a formal definition of relevance and its application to questions. In his analysis, questions are inherently underspecified or vague (1): their denotation contains a variable Op (1b) that depends on a decision problem facing the speaker. In the context of the decision problem, vagueness is resolved and a partition is chosen as the import of the question (1d).

(1) a. Where do you live?

b. $\{\lambda v[g \in Op(P)(v)]: w \in W \& g \in Op(P)(w)\}$, with $P = \lambda w \lambda x$ You live in x in w

c. *Possible partitions*: i. {You live in USA, You live in France, ... }

ii. {You live at 1 Lee St. Enid OK, You live at 2 3rd St. Ney OH, ... }

d. *Goal 1*: At LSA, questioner interested in background of hearer \rightarrow partition i

Goal 2: At LSA, questioner has to mail hearer a package \rightarrow partition ii

The DT approach derives two main types of vagueness: level of fine-grainedness as in (1), and degree of exhaustivity (2), where the desired answer could be strongly exhaustive (mention-all) or mention-some.

(2) Where can I buy an Italian newspaper?

Vagueness in plurals. Striking parallels exist between the types of vagueness inherent in questions and those in definite plurals. First, as repeatedly noted in the literature, definite plurals are vague with respect to the level of granularity/distributivity (3). Schwarzschild (1991) models this using a variable over covers (Cov_i), which allows VP to distribute up to sub-pluralities of the definite plural given by the cover (4b). Using *covers* (4a) and the specific cover (4b), Schwarzschild derives the intermediate distributivity reading (4c-i).

(3) a. The boys built a raft (, climbed a hill, and made a campfire)

b. $\forall x[x \in [[Cov_i]] \& x \subseteq [[the.boys']] \rightarrow x \subseteq [[built.a.raft']]]$

c. *Possible readings*: i. Team1={Andy,Bill} built a raft & Team2={Chris,Dan} built a raft

ii. Andy, Bill, Chris, and Dan each built a raft

d. *Scenario 1*: In a raft-building competition, a judge is deciding who to pass. \rightarrow i

Scenario 2: In the same competition, a judge is evaluating work-load on each child \rightarrow ii

(4) a. Cover: a set of sets of entities, s.t. the union of the sets in the cover is the universe of discourse.

b. $\{\{Andy, Bill\}, \{Chris, Dan\}, \{the\ coach... \}\}$

Second, definites are vague w.r.t. exhaustivity: (5) is compatible with the maximal (all the windows) or non-maximal construal of the definite, e.g. excepting a few closed windows if the hearer is wondering if his home is storm-proof (Krifka 1996).

(5) a. The windows are open. b. $\forall x[x \in [[Cov_i]] \& x \subseteq [[the.windows']] \rightarrow x \subseteq [[open']]]$

c. *Possible readings*: i. All of the windows are open ii. Some of the windows are open

d. *Scenario 1*: The house is being prepared for arrival of window-frame painters \rightarrow i

Scenario 2: Before a trip, hearer is wondering if the house is safe to leave \rightarrow ii

Brisson (1998) builds on Schwarzschild (1991) to permit exceptions by allowing mismatch between the distribution of individuals into the cover-cells and the NP denotations (ill-fitting covers (6a), cover in (6b) derives 5d:scenario2). Unlike Landman (1989) (who posits a pervasive ambiguity between singular and plural VPs and no mechanism for resolving the vagueness), she only derives cases of a few salient exceptional items, rather than existential readings like (5c-ii).

(6) a. A cover is *ill-fitting w.r.t. NP denotation* if some members of NP denotation are in the

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same covercell as non-members, so no union of cells in the cover equals the NP.

b. $\{\{window1\}, \{window2\}, \{window3, window4, window5, window6, Mary, door\}\}$

Both the “packaging” of entities within NP denotation and speaker/hearer goals play a role in the scenarios in (3,5); the cover account builds in the reference to packaging into the analysis. While in some cases (e.g.,3:scenario1)) such reference is necessary (otherwise semantics allowing the full range of readings is too weak), in other cases (e.g. 5:scenario2) it is wrong (non-maximality in this account is a matter of domain selection, so non-maximal reading for (5) is essentially ‘The salient/relevant subset of the windows is open.’ However, there is no ‘relevant set’ of windows – no window in scenario 2 is more relevant/salient than another.) Also, no existing proposal for definite plurals spells out the role of speaker/hearer goals.

The proposal. I apply van Rooy’s DT-based definition of relevance to definite plurals, replacing the cover analysis. I integrate Landman’s (1989) theory of groups with a principled account of how and when the various readings arise. I assume a model of discourse in which each utterance is associated with a speaker intention or goal (Grosz and Sidner 1986). When interpreting a vague utterance with a definite plural, hearers select propositions which can influence hearers’ actions in achieving the goal. Agents’ intentions can be represented as decision problems (DeP) they are solving. A DeP is a triple $\langle P, U, A \rangle$, where the probability function P represents agent’s beliefs, utility function U reflects the agent’s goals, and a set of (mutually exclusive) actions A the agent chooses from. A proposition q changes agent’s beliefs (P), resolving the DeP if, after q is learned, a single action has, in each resulting world, the highest utility.

In making an utterance, the speaker aims to resolve hearers’ DeP. A relevance ordering between propositions (7) yields the contextual criterion for licensing and choosing an interpretation for plural definites, just as it does for questions, with relevance defined as helpfulness in resolving the DeP.

(7) Proposition p is more relevant (better to learn) for resolving DeP than proposition q ($p >_{DeP} q$) iff

- i. p eliminates more actions as non-optimal than q does or
- ii. p eliminates the same number of actions as q does, and q entails p (i.e. q is over-informative)

This relevance ordering is built into the definition of the variable REL (for ‘relevant,’ instead of van Rooy’s Op) (8, where $*Ag(e) = \sqcup_{e' \preceq e} Ag(e')$). (9) gives the schema for the whole sentence; REL replaces Schwarzschild’s (1991) Cov_i in encoding the vagueness.

(8) $REL(DeP)(NP)(VP) = \lambda w. \{g(w): \sqcup\{(\downarrow d): d \in AT(g(w))\} \subseteq \downarrow NP(w) \ \&$

$\neg \exists h [\sqcup\{(\downarrow d): d \in AT(h(w))\} \subseteq \downarrow NP(w) \ \&$

$\exists e VP(e) \ \& h(w) = *Ag(e) >_{DeP} \exists e VP(e) \ \& g(w) = *Ag(e) \}$

(9) $\lambda w. \exists p [p \in \{\lambda v [g(v) \in REL(DeP)(NP)(VP)(v) \ \& \exists e. VP(e) \ \& g(w) = *Ag(e)]: g(w) \in REL(DeP)(NP)(VP)(w) \ \& p(w) = 1]$

(10) illustrates how (7) allows hearers to choose an appropriate reading for the vague definite. (10) a. *DeP*: Before a thunderstorm, Hearer has to decide whether to go on with daily business (*Action1*) if all windows are closed) or return home (*Action2*) if some windows are open.

b. The windows are closed = $\lambda w. \exists p [p \in \{g(w) \in REL(DeP)(win)(clos)(w) \ \& g(w) \text{ is closed} \} \ \& p = 1]$

☺ i. $\lambda w. \exists p [p \in \{\exists e. *clos(e) \ \& w1 + w2 + w3 + w4 + w5 = *Ag(e) \} \ \& p = 1]$ (all the windows...)

ii. $\lambda w. \exists p [p \in \{\exists e. *clos(e) \ \& w1 = *Ag(e), \exists e. *clos(e) \ \& w2 = *Ag(e), \dots \} \ \& p = 1]$ (some windows...)

c. The windows are open = $\lambda w. \exists p [p \in \{g(w) \in REL(DeP)(win)(open)(w) \ \& g(w) \text{ is open} \} \ \& p = 1]$

i. $\lambda w. \exists p [p \in \{\exists e. *open(e) \ \& w1 + w2 + w3 + w4 + w5 = *Ag(e) \} \ \& p = 1]$ (all the windows...)

© ii. $\lambda w. \exists p [p \in \{ \exists e. *open(e) \& w1 = *Ag(e), \exists e. *open(e) \& w2 = *Ag(e), \dots \} \& p = 1]$ (**some windows...**)

Consider an agent facing the problem in (10a). If he hears (10b), he has to select one of the interpretations [i] or [ii] as the true import of the vague literal statement. While [i] resolves the decision problem (by pointing to *Action1*), [ii] fails to do so, since it is compatible both with a scenario where all windows are closed, and one in which some are open and some closed. So, [i] is the only relevant interpretation. If the agent hears (10c) instead, both (c-i) and (c-ii) resolve the DeP (by pointing to *Action2*) and thus are both relevant. But [i] entails [ii] and thus is over-informative (hence less relevant according to (7)). This points to [ii] as the intended message.

Similarly, for the sentence (3a,11), hearers choose the level of distributivity based on (7).

(11) a. *De P*: Hearer is deciding whether to pass the various boys' teams in a raft-building competition; Andy, Bill, Chris, and Dan are all the boys. *Action1*: pass Team1 & fail Team2.

Action2: pass Team2 & fail Team1. *Action3*: pass both, *Action4*: fail both.

b. The boys built a raft = $\lambda w. \exists p [p \in \{ g(w) \in REL(DeP)(boys)(built.raft)(w) \& g(w) \text{ built a raft} \} \& p = 1]$

i. Andy and Bill and Chris and Dan (all the boys) collectively built a raft.

☺ ii. Team 1 built a raft & team 2 built a raft.

iii. Andy built a raft & Bill built a raft & Chris built a raft & Dan built a raft.

Interpretation [i] fails to resolve the question of the teams' performance. Both interpretations [ii] and [iii] resolve the problem, and so are relevant. But [iii] is over-informative, and so [ii] is the interpretation of choice.

Overt operators. Not all sentences with definite plurals are vague: e.g., in (12), *all* and *each* force fully maximal and fully distributive interpretations, respectively, compare with (13).

(12) a. All the boys surrounded the castle. b. The boys built a raft each.

(13) a. The boys surrounded the castle. b. The boys built a raft.

Theoretically, two mechanisms are possible for achieving this effect. First, the lexical operator (*all* or *each*) can work in the semantics to change the available readings. Another possibility is for these operators to contribute information about extra-linguistic factors (affecting the Decision Problem), so that REL, as a result, will give the desired readings. Winter (1998, 1999, 2001) adopts the former approach to these operators, while Brisson (1998) pursues the latter. The data suggests that no extra-linguistic factors can force a maximal reading in collective sentences like (12a). This is matched by inability of REL, on our account, to distinguish between maximal and non-maximal readings for collective predication. Thus, I will adopt Winter's approach, in which *all* (and *each*) work in the semantics to introduce universal quantification (and full distributivity) over atoms in the NP.

Conclusion. The DT approach structures the domain for definite plurals based on the notion of relevance rooted in the speaker's communicative goals, deriving the (non)-maximality and distributivity patterns while making correct predictions regarding speaker/hearer knowledge and spelling out the role of context in the interpretation of definite plurals.

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