Towards a Montagovian account of dynamics

— Abstract —

Introduction It is almost a truism that the interpretation of discourse is a dynamic process. In particular, when trying to give a logical semantics to a discourse, one faces the problem that quantifiers should dynamically extend their scope from one sentence to another. This problem is solved in Kamp's DRT[1] by using sets of reference markers. These, from an intuitive point of view, act as existential quantifiers. From a technical point of view, however, they must be considered as free variables, which explains that they can indeed extend their scope dynamically.

Because of this hybrid status, one has to choose carefully the names of the new reference markers when constructing a DRS. This problem is particularly salient when trying to combine DRT and Montague semantics[2] because the merging of two DRSs may result in reference marker assignments that destroy the values previously assigned. As a consequence, merging two DRSs is either a partial operation or must include careful variable renaming in order to avoid unwanted side effects.[3]

In this paper, we tackle this problem by providing Montague semantics with an appropriate notion of context that allow for some kind of dynamics without violating the usual quantifier scoping rules. In particular, the notions of free and bound variables are as usual, and the operation corresponding to the merging of two DRSs may be performed by standard β-reduction.

Expressing propositions in context Our proposal is based on the two following ideas: (i) we will interpret a sentence according to both its left and right contexts; (ii) these two kind of contexts will be abstracted over the meaning of the sentences.

Montague semantics[4] is based on Church's simple type theory,[5] which provides a full hierarchy of functional types built upon two atomic types: ι, the type of individuals, and o, the type of propositions. In order to accommodate a notion of context, we add a third atomic type, namely, γ. This type stands for the type of the left contexts. Then, if we only consider discourses made of declarative sentences, a right context is a piece of discourse that will be interpreted as a proposition provided it is given its left context. Consequently, the type of the right contexts must be $\gamma \rightarrow o$. Such a type may be seen as the type of a *continuation*,[6] which is indeed a standard way of modeling right contexts in

[1] H. Kamp and U. Reyle. *From Discourse to Logic.* Kluwer Academic Publishers, Dordrecht, 1993.

[2] J. Groenendijk and M. Stokhof. Dynamic Montague grammar. Technical Report LP-90-02, Faculty of Mathematics and Computer Science, Amsterdam, 1990.

R. Muskens. Combining Montague semantics and discourse representation. *Linguistics and Philosophy*, 19:143–186, 1995.

[3] J. van Eijck and H. Kamp. Representing discourse in context. In J. van Benthem and A. ter Meulen, editors, *Handbook of Logic and Language*, chapter 3. Elsevier, 1997.

[4] R. Montague. The proper treatment of quantification in ordinary english. In J. Hintikka, J. Moravcsik, and P. Suppes, editors, *Approaches to natural language: proceedings of the 1970 Stanford workshop on Grammar and Semantics*, Dordrecht, 1973. Reidel.

[5] A. Church. A formulation of the simple theory of types. *Journal of Symbolic Logic*, 5:56–68, 1940.

[6] C. Stratchey and C. Wadsworth. Continuations a mathematical semantics for handling full jumps. Technical Report PRG-11, Oxford University, Computing Laboratory, 1974.

denotational semantics.

Now, let s be the syntactic category of sentences, and t be the syntactic category of texts (or discourses). As we said, we intend to abstract over the meaning of a sentence its left and right contexts. Consequently, we obtain the following semantic interpretation for s and t:

$$[\![s]\!] = [\![t]\!] = \gamma \to (\gamma \to o) \to o \tag{1}$$

An example We do not have enough space in this abstract to develop the technicalities of our proposal. We just illustrate our approach by providing a solution to the famous donkey-sentence puzzle:

Every[1] farmer who owns a[2] donkey beats it$_2$.

For the sake of simplicity, we asume that the anaphoric references are explicitely given by some indexing, in which case the elements of type γ may be defined as simple association lists that are updated and accessed using two operators, namely, `push` and `sel`. It is important to stress, however, that our approach does not rely on this hypothesis. In the present example, `push` and `sel` must be considered as idealized context-updating and anaphora-resolution operators.

Let n and np be the syntactic categories of nouns and noun phrase, respectively. In order to obtain their semantic interpretation, we simply propagate Equation 1 in Montague's original interpretations:

$$[\![n]\!] = \iota \to [\![s]\!] \tag{2}$$
$$[\![np]\!] = (\iota \to [\![s]\!]) \to [\![s]\!] \tag{3}$$

Spelling it out, this yields the following semantic equations:

$$[\![n]\!] = \iota \to \gamma \to (\gamma \to o) \to o \tag{4}$$
$$[\![np]\!] = (\iota \to \gamma \to (\gamma \to o) \to o) \to \gamma \to (\gamma \to o) \to o \tag{5}$$

The following toy grammar is consistent with these interpretations:

expression	category	interpretation
farmer	n	$\lambda x e \phi.\, \mathbf{farmer}\, x \wedge \phi\, e$
donkey	n	$\lambda x e \phi.\, \mathbf{donkey}\, x \wedge \phi\, e$
owns	$np \to np \to s$	$\lambda o s.\, s\, (\lambda x.\, o\, (\lambda y e \phi.\, \mathbf{own}\, x\, y \wedge \phi\, e))$
beats	$np \to np \to s$	$\lambda o s.\, s\, (\lambda x.\, o\, (\lambda y e \phi.\, \mathbf{beat}\, x\, y \wedge \phi\, e))$
who	$(np \to s) \to n \to n$	$\lambda r n x e \phi.\, n\, x\, e\, (\lambda e.\, r\, (\lambda \psi.\, \psi\, x)\, e\, \phi)$
a^i	$n \to np$	$\lambda n \psi e \phi.\, \exists x.\, n\, x\, e\, (\lambda e.\, \psi\, x\, (\mathtt{push}\, i\, x\, e)\, \phi)$
every^i	$n \to np$	$\lambda n \psi e \phi.\, (\forall x.\, \neg(n\, x\, e\, (\lambda e.\, \neg(\psi\, x\, (\mathtt{push}\, i\, x\, e)\, (\lambda e.\, \top))))) \wedge \phi\, e$
it$_i$	np	$\lambda \psi e \phi.\, \psi\, (\mathtt{sel}\, i\, e)\, e\, \phi$

According to this grammar, we have that the following λ-term:

$$[\![\text{beats}]\!]\, [\![\text{it}_2]\!]\, ([\![\text{every}^1]\!]\, ([\![\text{who}]\!]\, (\lambda s.\, [\![\text{owns}]\!]\, ([\![\text{a}^2]\!]\, [\![\text{donkey}]\!])\, s)\, [\![\text{farmer}]\!]))$$

β-reduces to a form which is logically equivalent to the following:

$$\lambda e \phi.\, (\forall x.\, \mathbf{farmer}\, x \supset (\forall y.\, (\mathbf{donkey}\, y \wedge \mathbf{own}\, x\, y) \supset \mathbf{beat}\, x\, y)) \wedge \phi\, e$$