リスト

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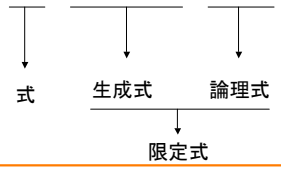
リストの表記法

- リスト：線形に順序のついた同じ型の値の集まり
  - [1,2,3] :: [Int]
  - [‘h’,’e’,’l’,’l’,’o’] :: [Char]
  - [[1,2],[3]] :: [[Int]]
  - [(+),(-)] :: [Int→Int→Int]
  - [ ] :: [a]
  - [1,3..5] :: [Int]
  - [1..] :: [Int]
  - [1,”fine day”]　X

リストの内包表記

- 集合を記述する数学の形式を元にした構文：

  [x*x | x <- [1..10], even x]



内包表記の例

- [(a,b) | a<-[1..3], b<-[1..2]]
  → [(1,1),(1,2),(2,1),(2,2),(3,1),(3,2)]
- [(i,j) | i<-[1..4], j<-[i+1..4]]
  → [(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)]
- [(i,j) | i<-[1..4], even i,j<-[i+1..4],odd j]
  → [(2,3)]
- [3 | j<-[1..4]]
  → [3,3,3,3]
- [‘ ‘ | j <- [1..5]]
  → “　“

例題

- 正の整数の約数のリストを生成する関数

  divisors n = [d | d<-[1..n], n `mod` d==0]

- 2つの正整数の最大公約数を求める関数

  gcd a b = maximum [d | d <- divisors a,
  b `mod` d == 0]

- 素数を判定する関数

  prime n = (divisors n == [1,n])

例題: Pythagoras数

- 与えられた範囲内のx^2+y^2=z^2のすべて（本質的違うような）x,y,zを求める関数

  triads n = [(x,y,z) | x<-[1..n],
  y<-[x..n],
  z<-[y..n],
  x^2+y^2==z^2]

練習問題

- 次の式を評価せよ.
  [j | i <- [1,-1,2,-2], i>0, j <- [1..i]]
- 式 (divisors 0)の値を決定せよ.
  divisors n = [d | d <- [1..n], n `mod` d == 0]
- 関数 intpairs を定義し, (intpairs n) が異なるすべての整数 1 <= x, y <= n の組のリストを生成するようにせよ.

リストの演算

- リストの連接
  [1,2,3] ++ [4,5] ➔ [1,2,3,4,5]
  [1,2] ++ [] ++ [1] ➔ [1,2,1]
  – (++) :: [a] → [a] → [a]
  – 性質:
    - 結合的: (xs++ys)++zs = xs++(ys++zs)
    - 単位元: [] ++ xs = xs ++ [] = xs
  – concat :: [[a]] → [a]
    concat xss = [x | xs<-xss, x<-xs]
    例: concat [[1,2], [], [3,2,1]] ➔ [1,2,3,2,1]

練習問題

- 次の等式のうちで真のものはどれか.
  [[]] ++ xs == xs
  [[]] ++xs == [xs]
  [[]] ++ xs == [[],xs]
  [[]] ++ [xs] == [[],xs]
  [xs] ++ [] == [xs]
  [xs] ++ [ys] == [xs,ys]

リスト上の関数

- リストの長さ
  – length :: [a] -> Int
  – length [1,2,3] ➔ 3
  – length [] ➔ 0
  – 性質 length (xs++ys) = length xs + length ys
- リストの先頭要素と後部
  – head :: [a] -> a tail :: [a] -> [a]
  – head [1,2,3] ➔ 1 head [] = ⊥
  – tail [1,2,3] ➔ [2,3] tail [] = ⊥
  – 性質 xs = [head xs] ++ tail xs

リスト上の関数

- リストの前部と末尾要素
  – init :: [a] -> [a] last :: [a] -> a
  – init [1,2,3] ➔ [1,2]
  – last [1,2,3] ➔ 3
  – 性質 xs = init xs ++ [last xs]

リスト上の関数

- 部分リストの取り出し
  – take :: Int -> [a] -> [a]
    take 3 [1..10] ➔ [1,2,3]
    take 3 [1,2] ➔ [1,2]

  – drop :: Int -> [a] -> [a]
    drop 3 [1..10] ➔ [4,5,6,7,8,9,10]

  – 性質1 take m . drop n = drop n . take (m+n)
  – 性質2 drop m . drop n = drop (m+n)

リスト上の関数

- 部分リストの取り出し
  - takeWhile :: (a -> Bool) -> [a] -> [a]
    takeWhile even [2,4,6,1,5,6] ➔ [2,4,6]
  - dropWhile :: (a -> Bool) -> [a] -> [a]
    dropWhile even [2,4,6,1,5,6] ➔ [1,5,6]
- リストの反転
  - reverse :: [a] -> [a]
    reverse [1,2,3,4] ➔ [4,3,2,1]
    reverse “hello” ➔ “olleh”

リスト上の関数

- リストの綴じ合わせ
  zip :: [a] -> [b] -> [(a,b)]
  zip [1..3] [‘a’,’b’,’c’] ➔ [(1,’a’),(2,’b’),(3,’c’)]
  zipWith :: (a->b->c) -> [a] -> [b] -> [c]
  zipWith f xs ys = [f x y | (x,y) <- zip xs ys]
  例（内積の計算）:
  sp (xs,ys) = sum [x*y | (x,y) <- zip xs ys]
  sp (xs,ys) = sum (zipWith (*) xs ys)
  例（位置の計算）
  position xs x = [i | (i,y) <- zip [0..length xs-1] xs, x==y]
  例（非減少列の判定）
  nondec xs = and [x<=y | (x,y) <- zip xs (tail xs)]

リスト上の関数

- リストの番号づけ
  (!!) :: [a] -> Int -> a
  [2,4,6,8] !! 2 ➔ 6

  例（非減少判定）
  nondec xs = and [xs!!k <= xs !! (k+1)
  | k <- [0 .. length xs – 2]]

リスト上の関数

- リストの差
  (\\) :: [a] -> [a] -> [a]
  [1,2,1,3,1,3] \\ [1,3] ➔ [2,1,1,3]
  (注：List.hsをloadする必要がある。)

  例（置換）
  permutation xs ys = (xs \\ ys == []) &&
  (ys \\ xs == []

高階関数map

- 関数mapは関数をリストのそれぞれの要素に適用する。
  - 定義：
    map :: (a -> b) -> [a] -> [b]
    map f xs = [f x | x<-xs]
  - 例：map square [1,2,3] ➔ [1,4,9]
    sum (map square [1..100]) ➔ 338350
  - 性質：
    map (f.g) = map f . map g
    map f (xs++ys) = map f xs ++ map f ys
    map f . concat = concat . map (map f)

高階関数filter

- 関数filterは述語pとリストxsを引数にとり、要素がpを満たすような部分リストを返す。
  - 定義：
    filter :: (a -> Bool) -> [a] -> [a]
    filter p xs = [x | x<-xs, p x]
  - 例：filter even [1,2,4,5,32] ➔ [2,4,32]
  - 性質：
    filter p . filter q = filter q . filter p
    filter p (xs++ys) = filter p xs ++ filter p ys
    filter p . concat = concat . map (filter p)

内包表記の翻訳

- 内包表記 ➔ map, filter での表記
- 規則
  1. [x | x<-xs] ➔ xs
  2. [f x | x <- xs] ➔ map f xs
  3. [e | x<-xs, p x, …]
     ➔ [e | x <- filter p xs, …]
  4. [e | x <-xs, y<-ys, …]
     ➔ concat [[e | y<-ys,…] | x<-xs]

翻訳の例 (1)

[1 | x <- xs]
➔[const 1 x | x <- xs]　　const k x = k
➔map (const 1)

[x*x | x <- xs, even x]
➔[x*x | x <- filter even xs]
➔[square x | x<-filter even xs]　square x = x*x
➔map square (filter even xs)

翻訳例 (2)

[x | xs <- xss, x <- xs]
➔ concat [[x | x <- xs] | xs <- xss]
➔ concat [xs | xs <- xss]
➔ concat xss

[(i,j) | i <- [1..n], j <- [i+1..n]]
➔ concat [[(i,j) | j <- [i+1..n]] | i <- [1..n]]
➔ concat [map (pair i) [i+1..n] | i <- [1..n]]
➔ concat (map mpair [1..n])

練習問題

- 関数 filter は concat と map を用いて次のように定義することができる.
  filter p = concat . map box
  　where box x = …
  関数 box の定義を述べよ.
- (map map)の型はなにか.

高階関数foldr

- 畳み込み関数foldはリストを他の種類の値に変えることが出来る。
  – 右側畳み込みfoldr
  foldr :: (a->b->b) -> b -> [a] -> b
  foldr (⊕) a [x1,x2,…,xn] = x1⊕(x2⊕(…(xn⊕a)))

```
s = a;
for ( i=n; i>=1; i-- ) {
    s = x[i] + s
}
```

例

- 簡単な例
  sum = foldr (+) 0
  product = foldr (*) 1
  concat = foldr (++) []
  and = foldr (&&) True
  or = foldr (||) False
- 少し複雑な例
  reverse = foldr postfix []
  　where postfix x xs = xs ++ [x]

  takewhile p = foldr oplus []
  　where x `oplus` xs = if p x then [x]++xs else []

高階関数foldl

– 左側畳み込みfoldl

foldl :: (b -> a -> b) -> b -> [a] -> b

foldl (⊕) a [x1,x2,…,xn] = (((a⊕x1)⊕x2)…⊕ xn)

```
s = a;
for ( i=1; 1<=n, i++ ) {
    s = s + x[i]
}
```

例

- length

```
length = foldl oneplus 0
    where n `oplus` x = n+1
```

- [xn,…,x0] ➔ xn*10^n + … + x0

```
pack xs = foldl oplus 0 xs
    where n `oplus` x = 10*n + x
```

畳み込み演算の性質

- 第一双対定理

演算子⊕とeが単位半群:

x ⊕ (y ⊕ z) = (x ⊕ y) ⊕ z

e ⊕ x = x ⊕ e = x

xsが有限リスト

foldr (⊕) e xs = foldl (⊕) e xs

畳み込み演算の性質

- 第二双対定理

x ⊕ (y ⊗ z) = (x ⊕ y) ⊗ z

x ⊕ e = e ⊗ x

xsが有限リスト

foldr (⊕) e xs = foldl (⊗) e xs

畳み込み演算の性質

- 第三双対定理

foldr (⊕) e xs = foldl (⊗) e (reverse xs)

where x ⊗ y = y ⊕ x

練習問題

- 関数 foldr と map が次の関係を持つ.

foldr (⊕) a . map f = foldr (⊗) a

ここで x ⊗ r = f x ⊕ r とする.

この関係から, 次のような法則を導け.

foldl (⊕) a . map f = foldl (⊗) a

where r ⊗ x = ???

注:map f . reverse = reverse . map f および双対定理の一つも用いよ.

空でないリストの畳み込み

- foldr1
  foldr1 (⊕) [x1,x2,…,xn] = x1⊕(x2⊕(…⊕xn))
- foldl1
  foldl1 (⊕) [x1,x2,…,xn] = ((x1⊕x2)…)⊕xn
- 例
  maximum xs = foldr1 max xs
  = foldl1 max xs

リストの走査関数scanl, scanr

- 左(右)側の走査
  scanl (⊕) a [x1,x2,…,xn]
  = [a,
  a ⊕ x1,
  (a ⊕ x1) ⊕ x2,
  …,
  ((a⊕x1)⊕x2)…⊕ xn]
- 例:
  – 累積和: scanl (+) 0 [12,3,4,5] ➔ [0,1,3,6,10,15]
  – 累乗積: scanl (*) 1 [1,2,3,4,5] ➔ [1,1,2,6,24,120]

リストのパターン

- リストの構成
  – 構成子 []
    • 空リストを生成する
  – 構成子 (:)
    • リストの新たら第一要素として新しい値を挿入する

1:2:3:4:[] ←➔ [1,2,3,4]

リスト上の関数の定義

```
null [] = True
null (x:xs) = False

length [] = 0
length (x:xs) = 1 + length xs

reverse [] = []
reverse (x:xs) = reverse xs ++ [x]
```