Disjunction of Causes and Disjunctive Cause: a Solution to the Paradox of *Conditio Sine Qua Non* using Minimal Abduction

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Abstract. We consider a problem of causality in legal reasoning. *Conditio sine qua non (c.s.q.n)* is a frequently used heuristics which determines a causality in legal reasoning. We argue that a paradox of *c.s.q.n.*is derived from a confusion between disjunction of causes and disjunctive cause and give a logical solution to the paradox using *minimal abduction*.

Keywords. causality, *conditio sine qua non*, abduction

1. Introduction

In legal reasoning, *Conditio sine qua non (c.s.q.n)* is a frequently used heuristics which determines a causality in legal reasoning. It means that we determine "A caused B" if "A had not happened, B would not have happened". However, it causes a paradox.

Consider the following case:

1. A put 120 mg of the poison P into C's coffee in order to kill C.
2. B put 120 mg of the same poison P into the same coffee to kill C without knowing A put the same poison.
3. C drank the coffee and died.

In this case, according to the principle of *c.s.q.n.*, we cannot say that either of A or B's actions causes C's death since without either of actions, C would have died. This seems a paradox since the danger of C's death is much higher in the case than other case where A and B put 60 mg, but in the latter case, A and B are blamed by *c.s.q.n.*.

We attack this paradox by reformalizing the causality by *minimal abduction*. *Abduction* is a powerful logical tool to get an explanation or complement missing knowledge given observation. It has been widely used for various areas such as diagnosis, planning and natural language processing [4]. In abduction, one criterion for choosing better explanation among multiple explanations is *minimality*. The criterion is motivated from economy of reasoning such as Occam's razor and plausibility of the explanation (more information in explanation is used, less plausible these additional events are true).

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In this paper, we identify the source of the above paradox by formalizing *c.s.q.n.*in belief revision system [2]. We believe that that the source of the paradox is a confusion between *disjunction of causes* and *disjunctive cause*. The former means that "A is a cause" or "B is a cause" whereas the latter means that "A or B is a cause". We show that *c.s.q.n.*cannot express the former one, but confuse the former with the latter. Then, we show that minimal abduction can distinguish the former from the latter naturally.

There have been researches which use abduction in legal reasoning [3,8,9,6]. [3] suggests usage of an abductive system to find out a counterargument in legal argumentation. [8] and [9] formalize a similarity in cases using abduction. [6] uses abduction to reason about evidence in causal reasoning in legal case. However, as far as we know, there has been no research to relate abduction with *c.s.q.n.*.

2. Formalizing Conditio Sine Qua Non

Intuitive meaning of principle of *c.s.q.n.*is as follows:

> A fact is a cause of an observation if the fact had not happen, we would not have got the observation.

Firstly, we would like to formalize the principle in a logical way. Our proposal is using a formalization of *belief revision*.

We assume a classical propositional language to express the discourse. We assume that there is a background theory B which represents a causal relation between facts. Let C be a set of current contingent facts or events which are found to be true in the case. Let O be an observation of which we would like to identify the cause.

Firstly, we assume that $B \cup C \models O$. Here $\models$ means validity over formulas in usual sense. We would like to find a part of C which is most relevant to the cause of O. Since *c.s.q.n.*is expressed as counter-factual sentence, conditional logic would be suitable to represent the principle. However, in this paper, We use belief revision formalization here since it is easier to compare with our abductive formalization. This would be acceptable since we have shown that conditional logics has a correspondence with belief revision in [5].

We define the principle of *c.s.q.n.* in belief revision system as follows by defining a revision operator "$*$".

Definition 1 (Maximal Consistent Subset) *Let C be a set of formula. We say that C' is a maximal consistent subset of C if C' is a consistent subset of C and there is no consistent proper superset of C'.*

Definition 2 (Belief Revision Operator) *We define a belief revision operator $*$ as follows. Let T and P be a set of formulas. $T * P$ is the set of maximal consistent subsets of $T \cup P$ including P.*

In this operator, we firstly remove a minimal part of T which causes contradiction with P and then add P to the decreased set. This intuitively means that if we assume some counter-factuals P contrary to T, then we firstly delete effects of facts in T which contradicts P and then we add P. Since there might be more than one way to avoid contradiction, we have to consider the set of maximal consistent subsets.

Definition 3 (Causal Framework) *A* causal framework *is a triple $\langle B, C, O\rangle$ where B be a propositional theory, C be a set of propositional formulas and O be a propositional formula.*

We call B a *background theory*, C a set of *contingent facts* and O an *observation*.

Note that we allow contingent facts to be any formula including disjunctions since sometimes known facts in legal domain are represented in a disjunctive form.

Now, we define a *cause*, then define a *critical cause* as the logically strongest cause since we would like to find a necessary and sufficient cause of the observation.

Definition 4 (Cause in *c.s.q.n.*) *Let $\langle B, C, O\rangle$ be a causal framework. Let A be logical combination of any formulas in C using conjunction and disjunction. We say that A is a* cause *of O in* c.s.q.n. *if $B \cup C \models O$, but for any $S \in C * (B \cup \{\neg A\})$, $S \not\models O$.*

In the above definition, we restrict propositions occurring in A to those used in C since we would like to infer a cause in C.

Definition 5 (Critical Cause in *c.s.q.n.*) *Let $\langle B, C, O\rangle$ be a causal framework. Let A be a logical combination of any formulas in C using conjunction and disjunction. A is a* critical cause *of O in* c.s.q.n. *if there is no cause, A' such that $A' \models A$.*

The paradoxical example in Introduction section is formalized as follows.

Example 1 *Let $\langle B, C_1, O\rangle$ be a causal framework where B is the set of the following formulas:*

$$\{\ \mathtt{A_{120mg}} \supset \mathtt{C_{died}}.\quad \mathtt{B_{120mg}} \supset \mathtt{C_{died}}.\},$$

$C_1 = \{\mathtt{A_{120mg}}, \mathtt{B_{120mg}}\}$, and $O = C_{died}$. Then, $\mathtt{A_{120mg}}$ is not a cause of $\mathtt{C_{died}}$ in c.s.q.n. *since $B \cup C_1 \models \mathtt{C_{died}}$, and $C_1 * (B \cup \{\neg\mathtt{A_{120mg}}\}) = \{S_1\}$ where $S_1 = B \cup \{\neg\mathtt{A_{120mg}}, \mathtt{B_{120mg}}\}$, and $S_1 \models \mathtt{C_{died}}$. Similarly, $\mathtt{B_{120mg}}$ is not a cause of $\mathtt{C_{died}}$ in* c.s.q.n., *either.*

On the other hand, $A_{disj} = \mathtt{A_{120mg}} \vee \mathtt{B_{120mg}}$ is a cause of $\mathtt{C_{died}}$ in c.s.q.n. *since $C_1 * (B \cup \{\neg A_{disj}\}) = \{S_2\}$ where $S_2 = B \cup \{\neg\mathtt{A_{120mg}} \wedge \neg\mathtt{B_{120mg}}\}$, and $S_2 \not\models \mathtt{C_{died}}$. Moreover A_{disj} is the critical cause of $\mathtt{C_{died}}$ in* c.s.q.n..

This example actually have a parallel argument where the contingent fact is a disjunction.

1. There are two tablets one of which does not contain any poison, and the other of which contains 120 mg of poison P. A and B choose one of them to kill C and put both tablets into C's coffee.
2. C drank the coffee and died.

In this example, we cannot tell which of A or B killed C as follows.

Example 2 *Let $\langle B, C_2, O\rangle$ be a causal framework where B is the same as Example 1, $C_2 = \{\mathtt{A_{120mg}} \vee \mathtt{B_{120mg}}\}$, and $O = C_{died}$.*

$\mathtt{A_{120mg}}$ is not a cause of $\mathtt{C_{died}}$ in c.s.q.n. *since $B \cup C_2 \models \mathtt{C_{died}}$ and $C_2 * (B \cup \{\neg\mathtt{A_{120mg}}\}) = \{S_3\}$ where $S_3 = B \cup \{\neg\mathtt{A_{120mg}}, \mathtt{A_{120mg}} \vee \mathtt{B_{120mg}}\}$, and $S_3 \models \mathtt{C_{died}}$. Similarly, $\mathtt{B_{120mg}}$ is not a cause of $\mathtt{C_{died}}$ in* c.s.q.n., *either.*

On the other hand, $A_{disj} = \mathtt{A_{120mg}} \vee \mathtt{B_{120mg}}$ *is a cause of* $\mathtt{C_{died}}$ *in* c.s.q.n. *since* $C_2 * (B \cup \{\neg A_{disj})\}) = \{S_4\}$ *where* $S_4 = B \cup \{\neg \mathtt{A_{120mg}} \wedge \neg \mathtt{B_{120mg}}\}$*, and* $S_4 \not\models \mathtt{C_{died}}$*. Moreover, this is the critical cause of* $\mathtt{C_{died}}$ *in* c.s.q.n..

We believe that the above examples show a source of the paradox of *c.s.q.n.*. Basically there is a confusion between the paradoxical case and the disjunctive case. In the paradoxical case, although we could say $\mathtt{A_{120mg}}$ and $\mathtt{B_{120mg}}$ are independent cause of $\mathtt{C_{died}}$, we regard this as disjunctive cause $\mathtt{A_{120mg}} \vee \mathtt{B_{120mg}}$ which we cannot distinguish from such knowledge state that only disjunctive knowledge is known.

Our proposal is to distinguish the *disjunction of causes* from *disjunctive cause*. A similar issue has been long discussed in the context of the logic of knowledge, belief, and perception [1]. The ‘or’ in certain kinds of contexts is known to be decomposable. For example, “A knows that B or C won the race” does not imply “A knows that B won the race, or that C won the race.” Such clauses as are headed by the verbs of knowledge, belief, and perception may mention affairs in hypothetical worlds in the subjunctive mood. Thus, these verbs are often represented by modal operators that can access different possible worlds in Kripke semantics.

Given a modal operator K meaning ‘it is known that,’ we can represent the uncertainty of knowledge as $K(A_{120mg} \vee B_{120mg})$ and can distinguish it from the different kinds of disjunction, $KA_{120mg} \vee KB_{120mg}$ since

$$K(A_{120mg} \vee B_{120mg}) \not\supset KA_{120mg} \vee KB_{120mg}.$$

As causal reasoning in this paper is subjective, we should distinguish the statement “We believe that the cause of O is either A or B” from the statement “We believe that the cause of O is A or we believe that the cause of O is B”. Unfortunately, *c.s.q.n.* does not distinguish between the above statements, and even worse, *c.s.q.n.* cannot express the disjunction of causes.

Fortunately, there is a natural solution in the above, that is, *abduction*[2]. In abduction, we can naturally distinguish between above statements by considering disjunction of explanations and explanation represented as disjunction. Therefore, we can solve the paradox as the subsequent sections show.

3. Abductive Framework and Minimal Explanation

In this section, we define abduction.

Definition 6 (Abductive Framework) *An* abductive framework *is a triple* $\langle B, H, O \rangle$ *where* B *be a propositional theory,* H *be a set of propositional formulas and* O *be a propositional formula.*

We call B a *background theory*, H a set of *abducibles* and O an *observation*.

Definition 7 (Explanation) *Let* $\langle B, H, O \rangle$ *be an abductive framework.*

[2] Another solution would be employing a new modal operator representing causation to avoid the confusion between the disjunction of causes and the disjunctive cause. We leave this option as a further research.

- *A subset* E *of* H *is an* explanation *w.r.t.* $\langle B, H, O\rangle$ *if* $B \cup E \models O$ *and* $B \cup E \not\models false$.
- *An explanation* E *w.r.t.* $\langle B, H, O\rangle$ *is* minimal *if there exists no subset of* H*,* E' *s.t.* $E' \subset E$ *and* $B \cup E' \models O$*. ("*$\subset$*" is a strict subset relation)*

We denote all the minimal explanations w.r.t. $\langle B, H, O\rangle$ *as* $MinE_{B,H}(O)$*.*

If E is a minimal explanation, then if we remove any element of E, the resulting set does not explain the observation.

We call this reasoning of seeking minimal explanation as *minimal abduction*. Minimal abduction could be understood as seeking critical explanation which removes all the irrelevant parts of an explanation.

Example 3 *Consider the following abductive framework* $\langle B, H, O\rangle$ *where* B *is the set of the following formulas:*

$P \wedge Q \supset T. \quad P \wedge R \supset T. \quad P \wedge S \supset T.$

and $H = \{P, Q \vee S, R \vee S\}$ *and* $O ='' T''$*. Then,* $E_0 = \{P, Q \vee S, R \vee S\}$ *is an explanation w.r.t.* $\langle B, H, O\rangle$*, but not minimal since there are smaller explanations* $E_1 = \{P, Q \vee S\}$ *and* $E_2 = \{P, R \vee S\}$ *which is a proper subset of* E_0*. In this example,* $MinE_{B,H}(O) = \{E_1, E_2\}$*.*

4. Solution using Minimal Abduction

Let $\langle B, C, O\rangle$ be a causal framework. In order to define a new causal relationship, We translate it into an abductive $\langle B, H, O\rangle$ where $H = C$.

We define a new concept of cause in terms of abduction as follows.

Definition 8 (Minimal Cause in Abduction) *Let* $\langle B, C, O\rangle$ *be a causal framework and* $\langle B, H, O\rangle$ *be its translation of abductive framework. We define a* minimal cause in abduction *as an element of* $MinE_{B,H}(O)$*.*

Note that we no longer consider a logical combination of any formulas in C using conjunction and disjunction as a cause. That means that we do not consider any hypothesis which are derived from C using logical inference, but we consider only formulas explicitly mentioned in C. This is a unique feature of abduction which contribute a distinction between disjunctive hypothesis and disjunctions of hypotheses.

Example 4 *Consider the case in Example 1. In this example,* $H = \{\mathtt{A_{120mg}}, \mathtt{B_{120mg}}\}$*. Then,* $MinE_{B,H}(O)$ *becomes* $\{\{\mathtt{A_{120mg}}\}, \{\mathtt{B_{120mg}}\}\}$ *which means that the causes are a disjunction:* $\mathtt{A_{120mg}}$ *or* $\mathtt{B_{120mg}}$*.*

On the other hand, consider the case in Example 2. In this example, $H = \{\mathtt{A_{120mg}} \vee \mathtt{B_{120mg}}\}$*. Then,* $MinE_{B,H}(O)$ *becomes* $\{\{\mathtt{A_{120mg}} \vee \mathtt{B_{120mg}}\}\}$ *which means that the cause is a disjunctive cause:* $\mathtt{A_{120mg}} \vee \mathtt{B_{120mg}}$*.*
Therefore, we can distinguish between a disjunction of causes and a disjunctive cause naturally by minimal abduction.

There is correspondence between cause in *c.s.q.n.* and cause in minimal abduction as follows.

Theorem 1 (Relationship between *c.s.q.n.* and minimal abduction) *Let $\langle B, H, O\rangle$ be an abductive framework translated from a causal framework $\langle B, C, O\rangle$. Let $MinE_{B,H}(O) = \{E_1, ..., E_n\}$. Then, $conj(E_1) \vee ... \vee conj(E_n)$ is the critical cause in* c.s.q.n. *where $conj(E_i)$ is the conjunctions of elements in E_i.*

This correspondence could be understood as the correspondence in cautious reasoning and brave reasoning in Default Logic [7]. In cautious reasoning, we consider a formula which is common in all extensions in default theory whereas in brave reasoning, we consider a formula which exists in an extension.

So, our proposal using minimal abduction for causality could be regarded as a brave approach for causal reasoning in legal domain in which we consider a cause in each explanation in abduction. On the other hand, causality in *c.s.q.n.* could be regarded as a cautious approach in which a cause is a formula which is true in all explanations in abduction. However, since the distinction between the disjunction of cause and disjunctive cause should be made, we believe that the brave approach is more suitable for causal reasoning in legal domain.

5. Conclusion

We believe that contribution of this paper is as follows:

- We formalize the principle of *c.s.q.n.* in belief revision system and why *c.s.q.n.* causes a paradox.
- We give a solution by minimal abduction by distinguish the disjunction of causes and disjunctive cause.

We believe that some of paradoxes in legal reasoning could be solved by introducing careful analysis studied in knowledge reasoning and the semantics of natural language. Therefore, we would like to study legal reasoning toward this direction.

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