パターン認識
Pattern Recognition

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Final Report

- Find any pattern recognition paper in top journals or top conferences
- e.g., IEEE TPAMI, IJCV, CVPR, ICCV, NeurIPS, ICML, ACMMM...
- Describe the following:
  - bibliographic info of the paper
  - brief of the paper
  - what is the problem, why it's important, how it's solved, validation?
  - why you selected the paper, what is exciting?
  - feedback to the lecture, any comments
- (about) 2-4 pages A4
- due: 07/31/2023
- send via ITC-LMS

Final Report

- Geman and Geman, Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images, TPAMI, 1984.
- Moghaddam and Pentland, Probabilistic Visual Learning for Object Detection, TPAMI, 1997.
- Belhumeur, Hespanha, and Kriegman, Eigenfaces vs. Fisherfaces: Recognition Using Class Specific Linear Projection, TPAMI, 1997.
- Shi and Malik, Normalized Cuts and Image Segmentation, TPAMI, 2000.
- Belkin and Niyogi, Laplacian Eigenmaps and Spectral Techniques for Embedding and Clustering, NIPS, 2001.
- Zhang and Sim, When Fisher meets Fukunaga-Koontz: A New Look at Linear Discriminants, CVPR, 2006.
- Felzenszwalb, Girshick, McAllester, and Ramanan, Object Detection with Discriminatively Trained Part Based Models, TPAMI, 2009.
- Antonio Torralba and Alexei A. Efros, Unbiased look at dataset bias, CVPR 2011.

Clustering Experiments

Four types of dataset:

- Two Gaussians (two flattened Gaussians)
- Four squares
- Four Gaussians
- Swiss role

Three clustering algorithms:

- k-means
- Single linkage
- Complete linkage.

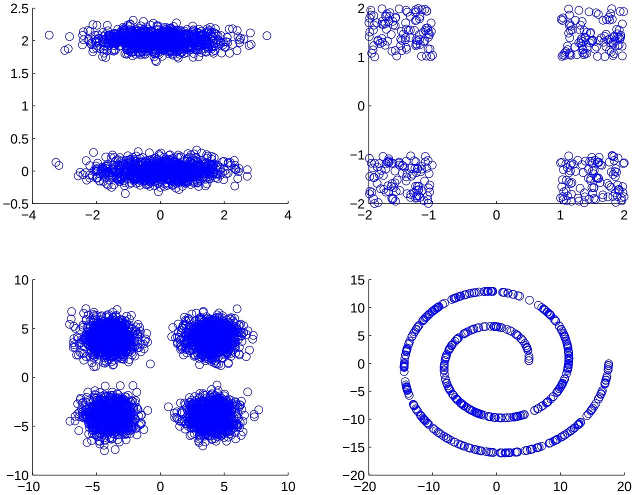
Clustering methods (recap)

k-means
: Select k representatives randomly, assign each data to its closest representative, and iteratively update representatives by their means of assigned data.

Single linkage
: Hierarchical aggromerative clustering method with cluster distance defined as minimum distance among items.

Complete linkage
: Hierarchical aggromerative clustering method with cluster distance defined as maximum distance among items.

Data



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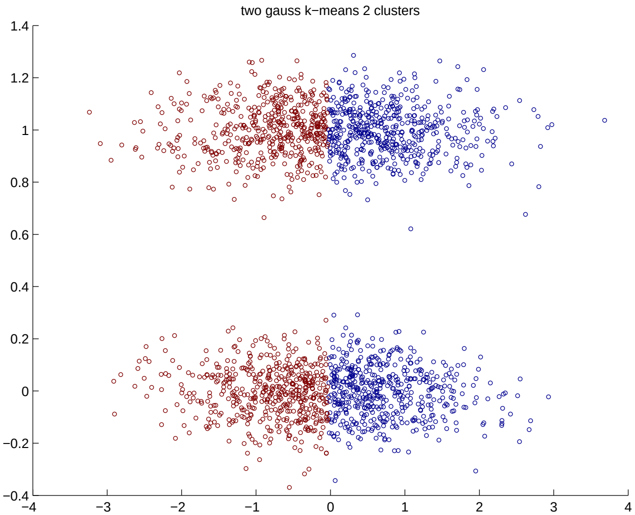
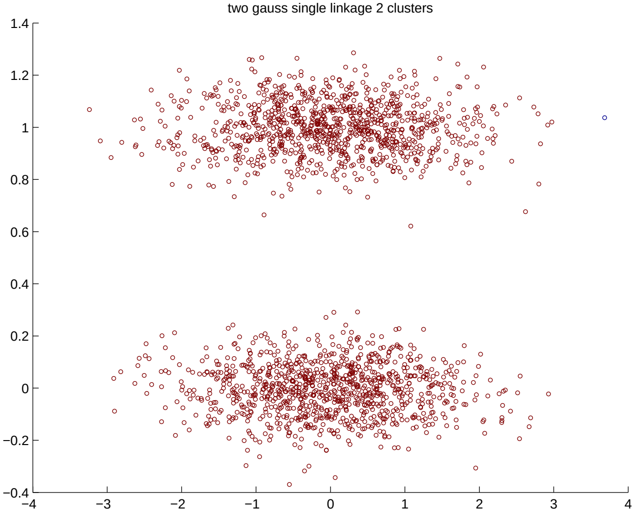
Two Gaussians



Two Gaussians



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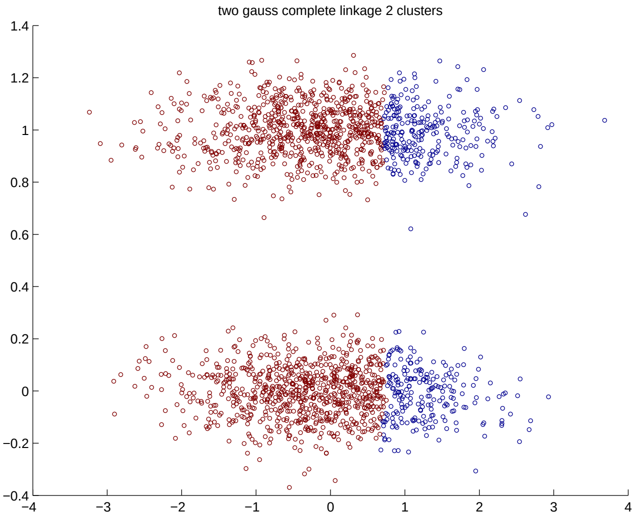

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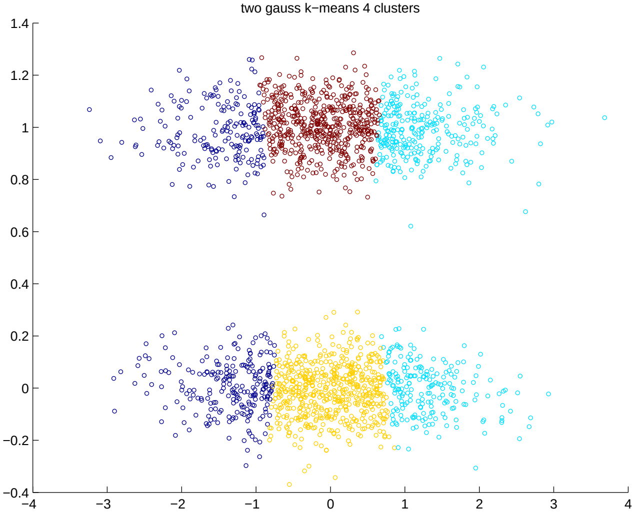

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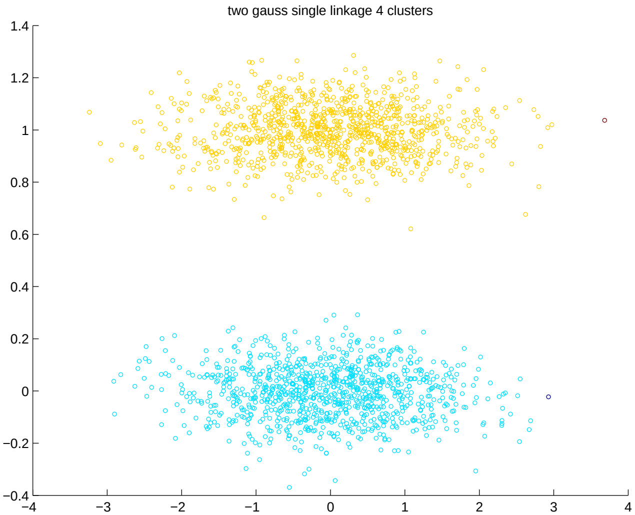

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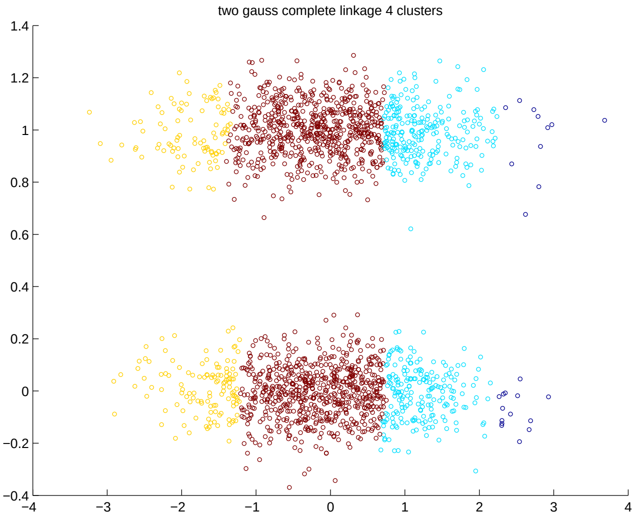

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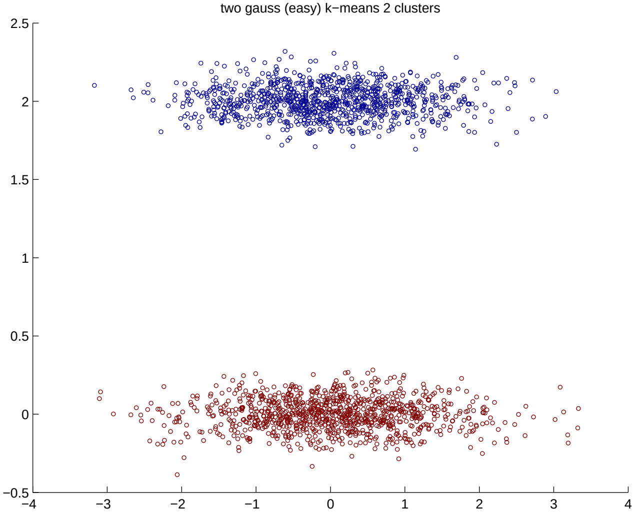

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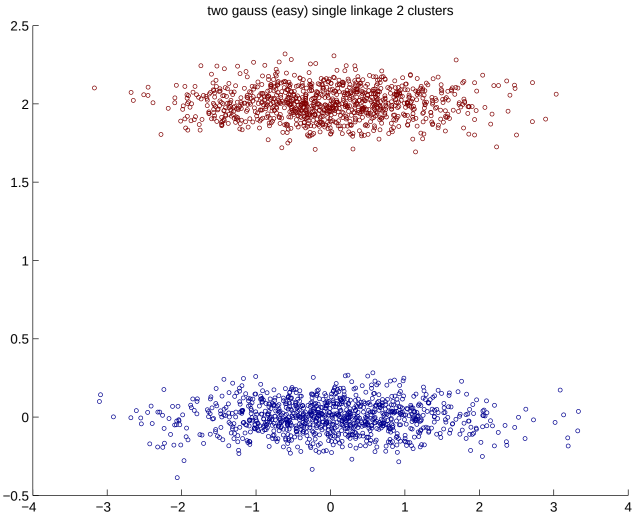

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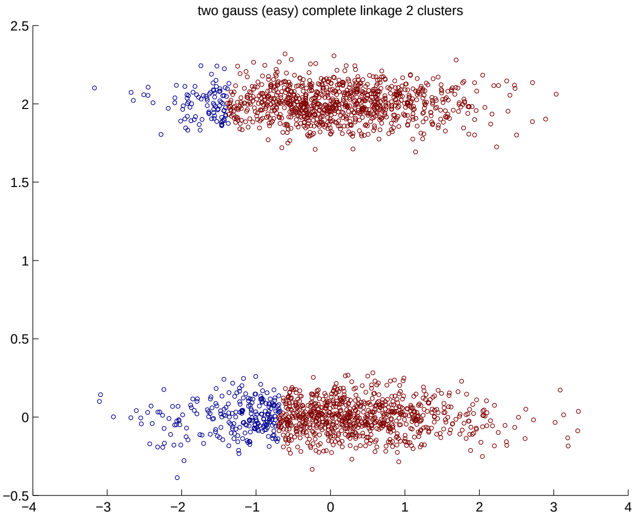
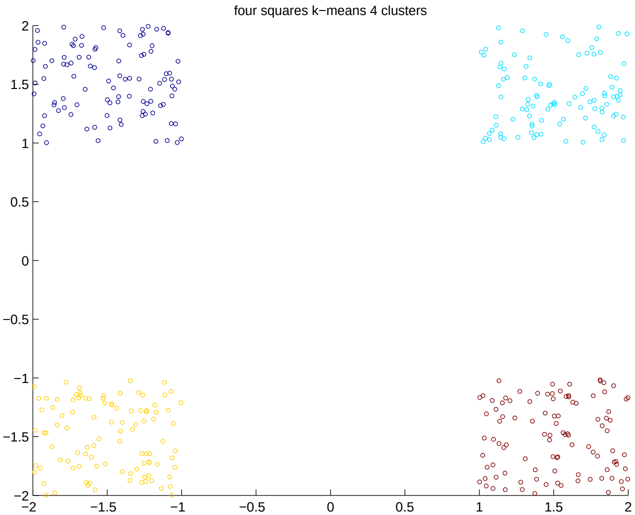


Four Squares



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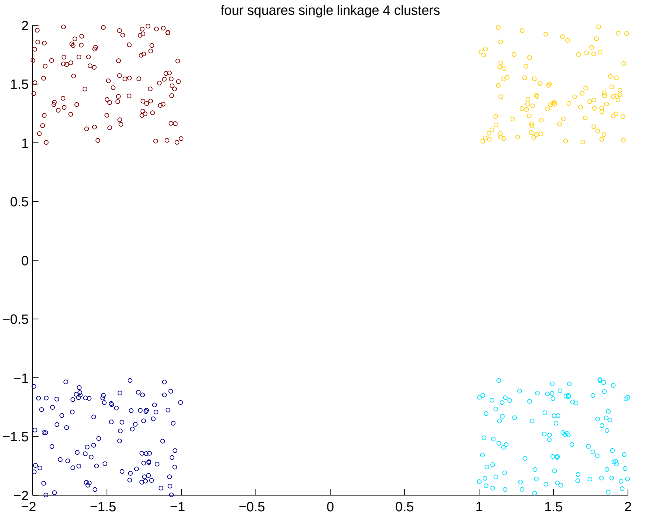
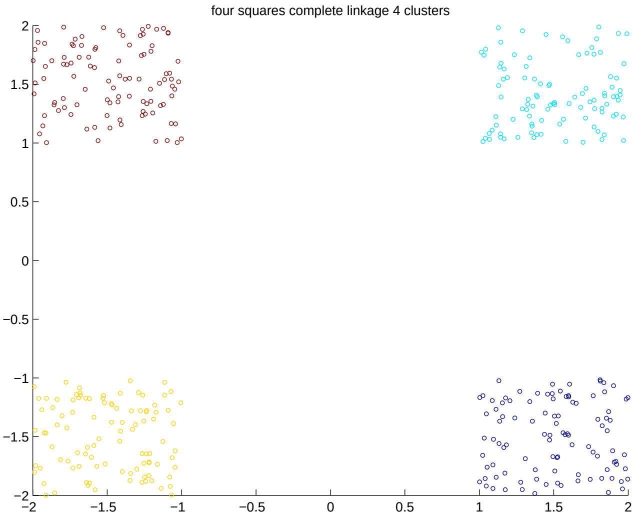
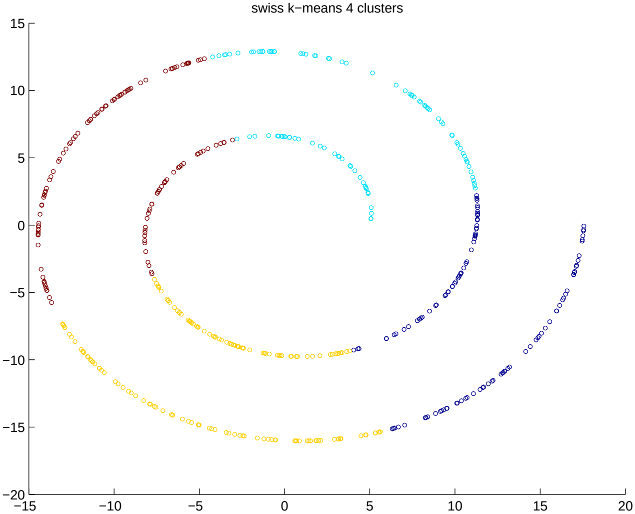


Four Squares



Swiss Role



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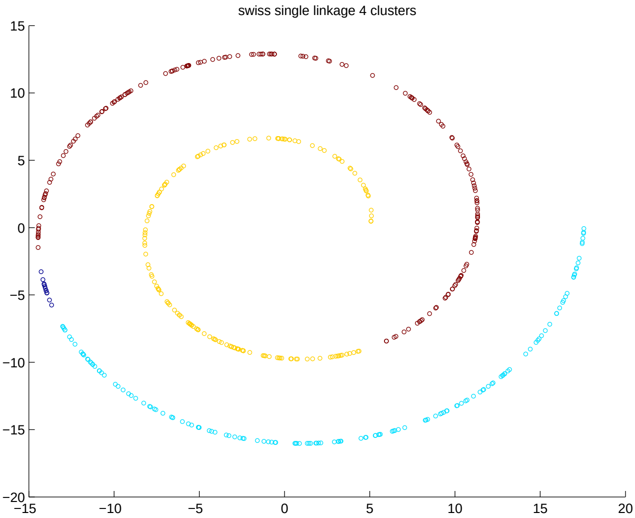

swiss single linkage 4 clusters
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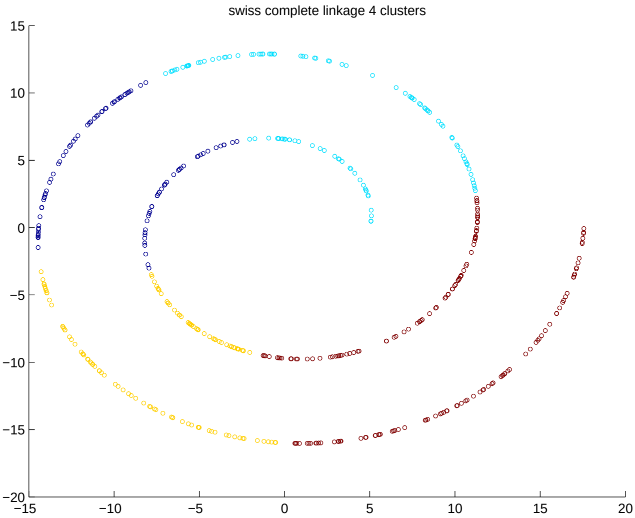
Swiss Role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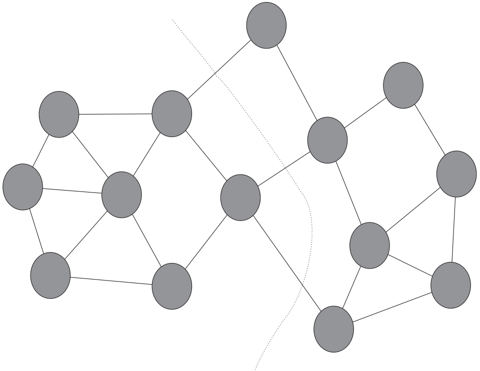
Graph Spectral Clustering

- So far we assume that data to be clustered have corresponding vectors, and distances can be evaluated as the Euclidean distances.
- But what if only distances (or similarities) are given?
- What if "distances" are effective in local region only?
- Graph-based clustering techniques may work in such situations.
- We represent data along with similarities as graph, and solve clustering as graph partitioning problem.

Graph Spectral Clustering

- Each data: node.
- Similarity between two nodes (if any): edge.
- Edge weight: similarity.
- Cut: sum of weights of edges which are cut.



Graph Spectral Clustering

Represent n-node graph as $n \times n$ matrix W where $w_{i,j}$ represents similarity between i-th and j-th nodes.
First we consider simple case to partition data into two sets.
We want to determine membership indicator q_i:

$$q_i = \begin{cases} 1 & \text{if } i \in A \\ -1 & \text{if } i \in B \end{cases}$$

$$\begin{aligned} J =& CutSize = \frac{1}{4} \sum_{i,j} w_{i,j} (q_i - q_j)^2 = \frac{1}{4} \sum_{i,j} w_{i,j} (q_i^2 + q_j^2 - 2 q_i q_j) \\ =& \frac{1}{2} \sum_{i,j} q_i (d_i \delta_{i,j} - w_{i,j}) q_j \\ =& \frac{1}{2} q^T (D - W) q \end{aligned}$$

$$d_i = \sum_j w_{i,j}$$

Minimization of $J = \frac{1}{2} q^T (D - W) q$ is hard (NP-complete).

So we relax q from discrete values to continuous values: then the solution of $\min J(q)$ can be obtained by the eigenvectors of

$$(D - W)q = \lambda q$$

Properties of Graph Laplacian

$L = D - W$ is called Laplacian matrix of the graph.
L is semi-positive definite: $x^T L x \geq 0$ for any x.
First eigenvector is $q_1 = [1\,1\,\cdots\,1]^T = e^T$ with $\lambda_1 = 0$.
Second eigenvector q_2 is the desired solution (called Fiedler vector).

$$\lambda_2 = \frac{cutsize}{|A|} + \frac{cutsize}{|B|}$$

Since J is insensitive to additive constant to q

$$J = \frac{1}{4} \sum_{i,j} w_{i,j}((q_i + c) - (q_j + c))^2$$

we sort q_2 and cut in the middle point.

Clustering Objective Functions

- Ratio Cut

$$J_{Rcut} = \frac{s(A,B)}{|A|} + \frac{s(A,B)}{|B|}$$

- Normalized Cut

$$\begin{aligned} J_{Ncut} &= \frac{s(A,B)}{d_A} + \frac{s(A,B)}{d_B} \\ &= \frac{s(A,B)}{s(A,A)+s(A,B)} + \frac{s(A,B)}{s(B,B)+s(A,B)} \end{aligned}$$

- Min-Max Cut

$$J_{MMcut} = \frac{s(A,B)}{s(A,A)} + \frac{s(A,B)}{s(B,B)}$$

$$s(A,B) = \sum_{i \in A} \sum_{j \in B} w_{i,j}, \;\; d(A) = \sum_{i \in A} d_i$$

Laplacian Eigenmaps

We embed low dimensional vector representation to each node.
To do so, we solve the following generalized eigenvalue problem:

$$Lf = \lambda Df.$$

(FYI the same solution with Normalized Cut.)
For $f_0, f_1, \ldots, f_n$ corresponding to $0 = \lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_n$,
we correspond $[f_1(i)\, f_2(i)\, \ldots\, f_m(i)]^T$ to the i-th node.

Experiment

- The weights of edges are defined as:

$$w_{i,j} = \begin{cases} e^{-\frac{d_{i,j}^2}{\sigma^2}} & \text{if } i \in knn(j) \text{ or } j \in knn(i) \\ 0 & otherwise \end{cases}$$

- Convert i-th node to $[f_1(i)\, f_2(i)\, \ldots\, f_m(i)]^T$ using Laplacian eigenmaps.
- Apply k-means to the converted vectors.
- Constants: k of knn: 5, σ: 10, m: 2.

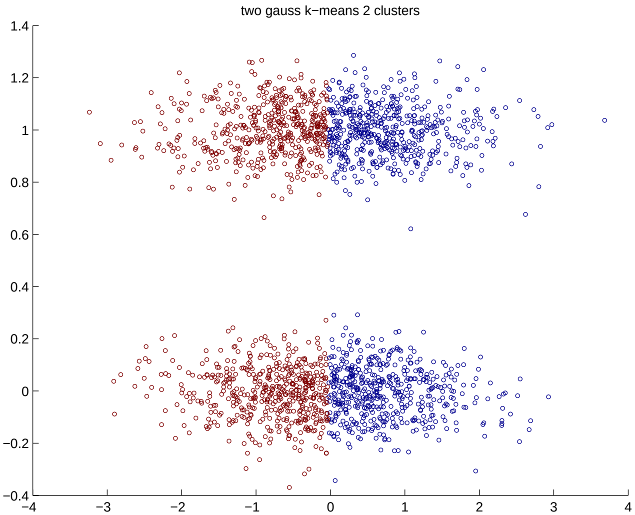
Laplacian Eigenmaps (Matlab)

```
k=10;
sigma=10;
n=size(x,1);
dt=squareform(pdist(x));
[sdt,idx]=sort(dt,'ascend');
dt=sdt(1:k+1,:);
nidx=idx(1:k+1,:);
tW=exp(-dt.^2/(sigma^2));
ii=repmat(1:n,k+1,1);
W=sparse(ii(:),double(nidx(:)),tW(:),n,n);
W=full(W);
W=max(W,W');
D=diag(sum(W,2));
[v,d]=eigs(D-W,D,10,'sa');
xx=v(:,2:3);
c=kmeans(xx,numc);
```

Laplacian Eigenmaps (Python)

```
k = 10
sigma = 5.
n = x.shape[0]
dt = squareform(pdist(x))
idx = np.argsort(dt)
dt = np.array([dt[i, idx[i, range(k)]] for i in range(n)])
W = np.zeros([n, n])
for i in range(n):
    for j in range(k):
        W[i, idx[i, j]] = np.exp(-(dt[i, j]**2) / (sigma**2))
W = np.maximum(W, W.T)
D = np.diag(np.sum(W, axis=0))
dd, v = scipy.sparse.linalg.eigs(D - W, k=5, M=D, which='SR')
v = np.real(v)
idxdd = np.argsort(np.real(dd))
xx = v[:, idxdd][:, range(1, 4)]
c = sklearn.cluster.KMeans(n_clusters=numc).fit_predict(xx)
```

Two Gaussians



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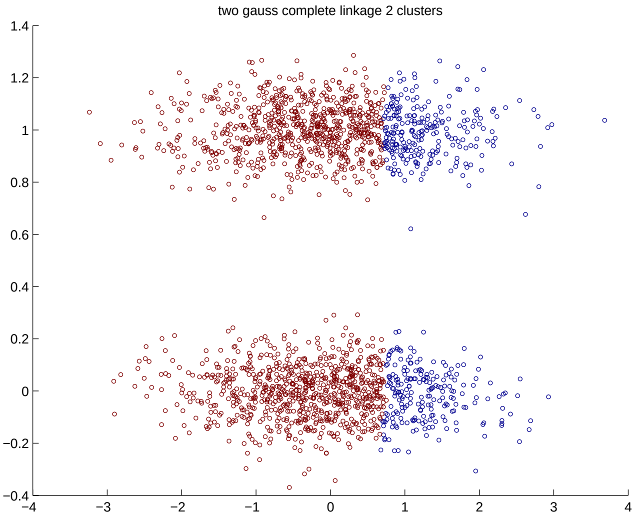

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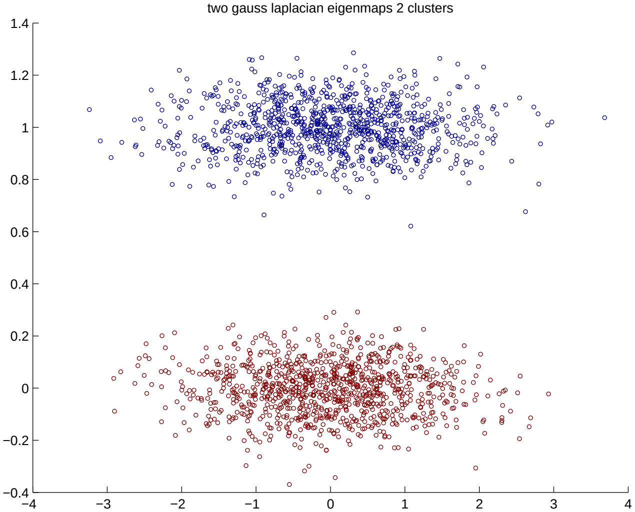

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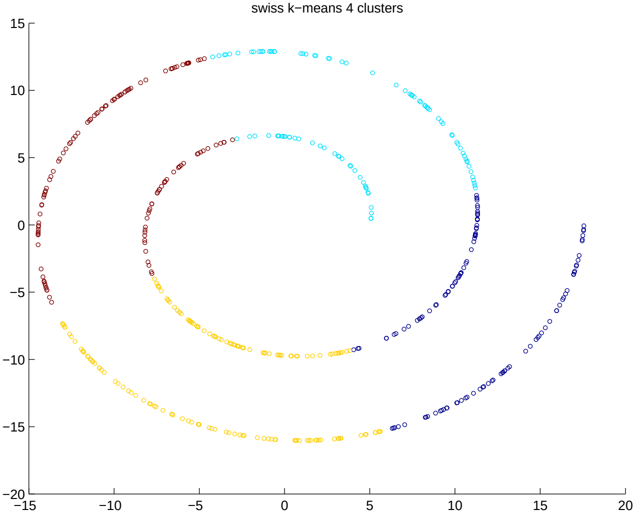
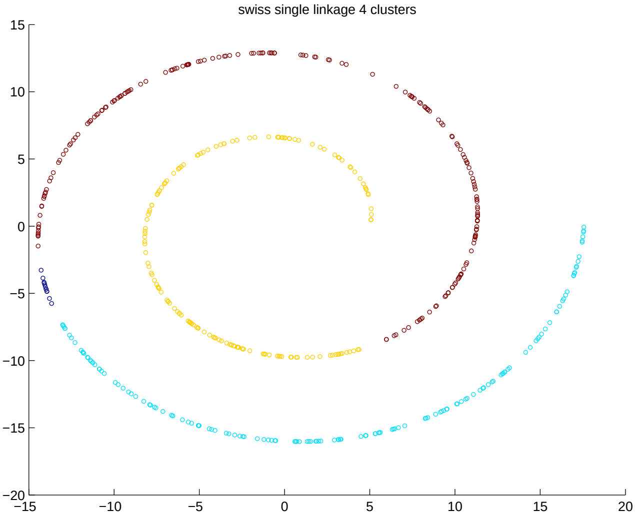


Swiss Role



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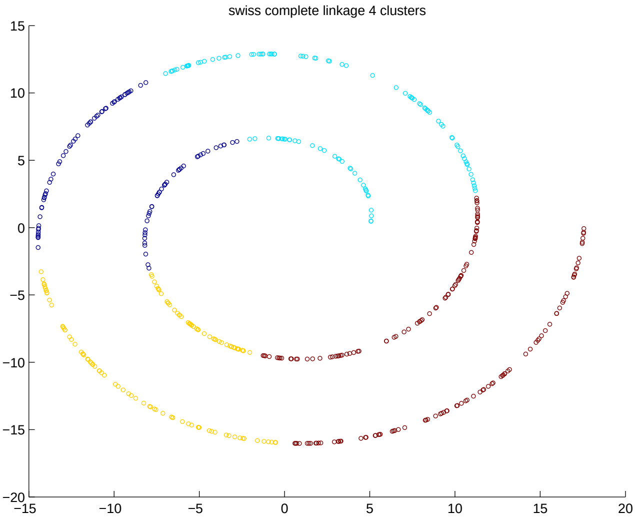
Swiss Role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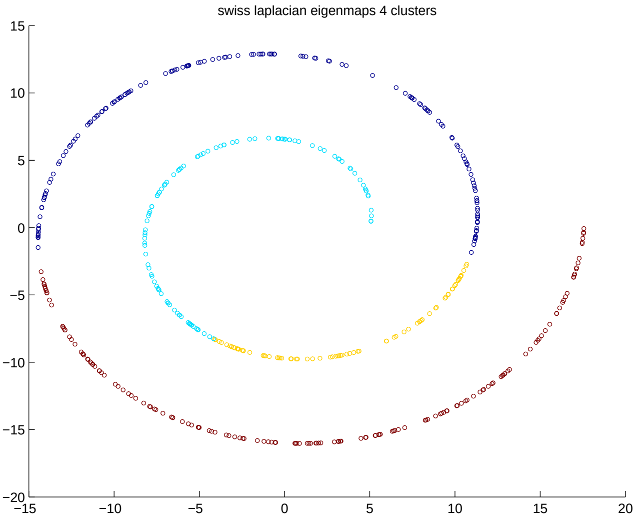

swiss laplacian eigenmaps 4 clusters
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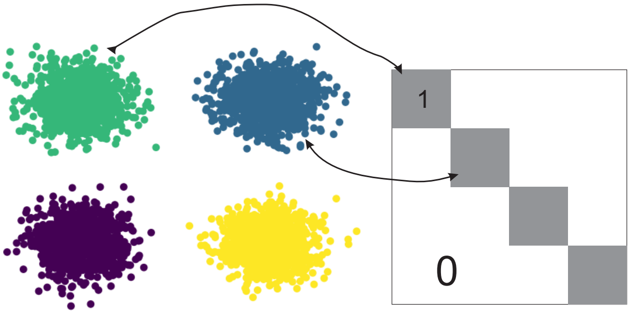
Example



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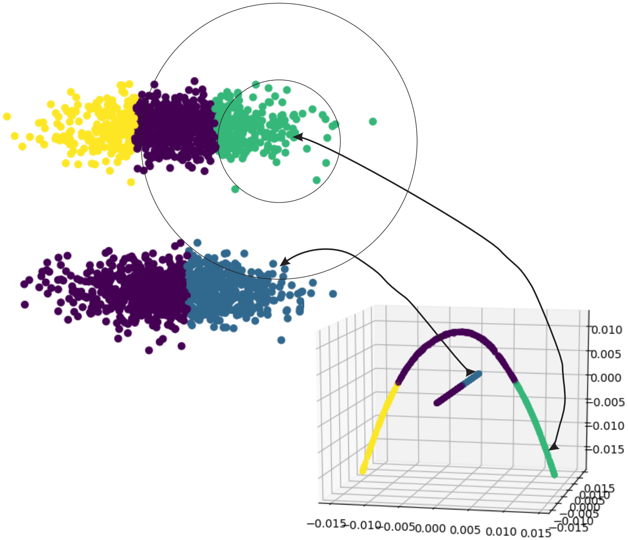
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