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ASPL-approximation for graph of diameter 3

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Problem

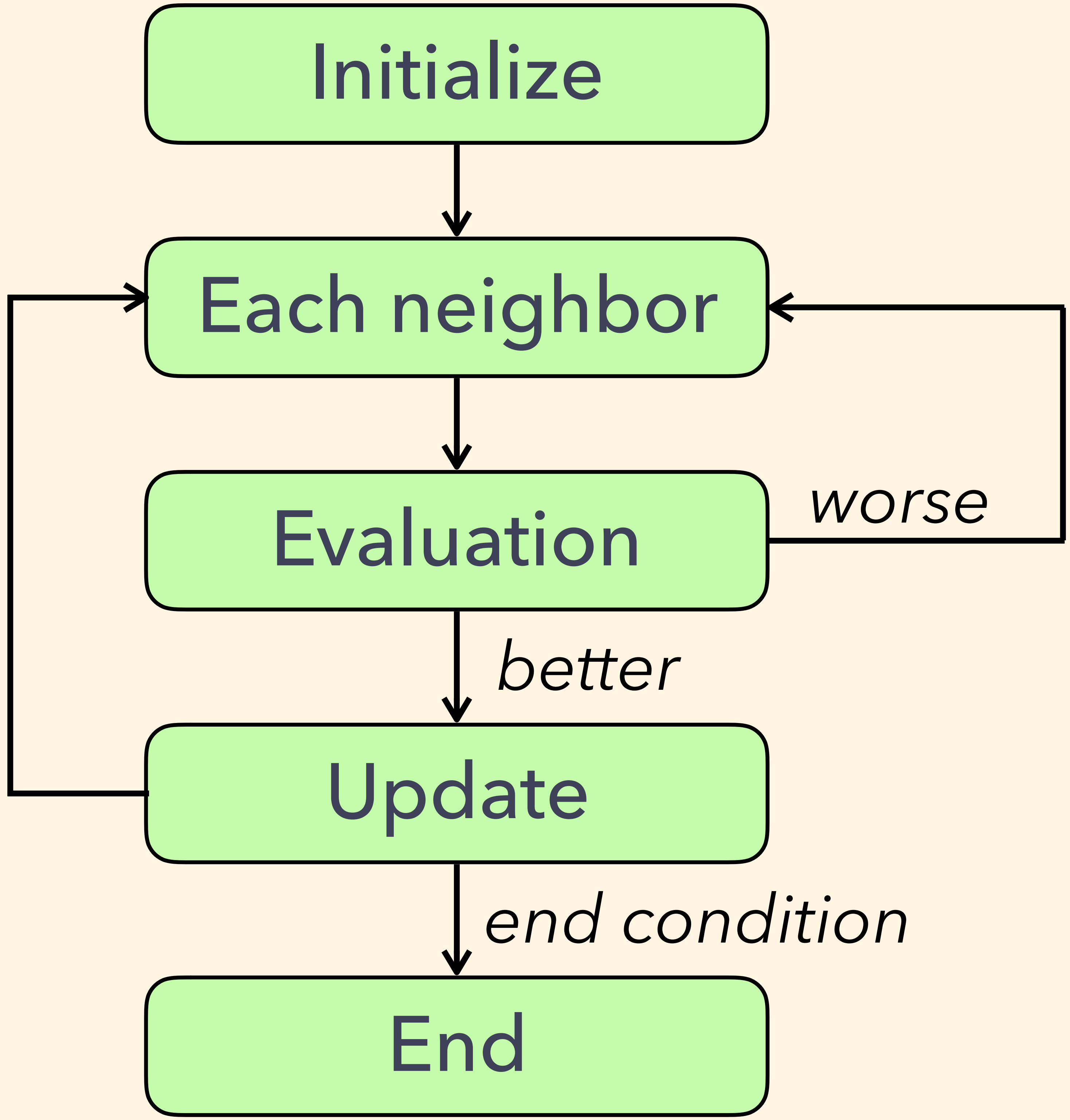
- ASPL : Average Shortest Path Length
- Given n and d, find low-ASPL graph of size n degree d.
- large n, large d → difficult
- **Small diameter → easier**

Main idea

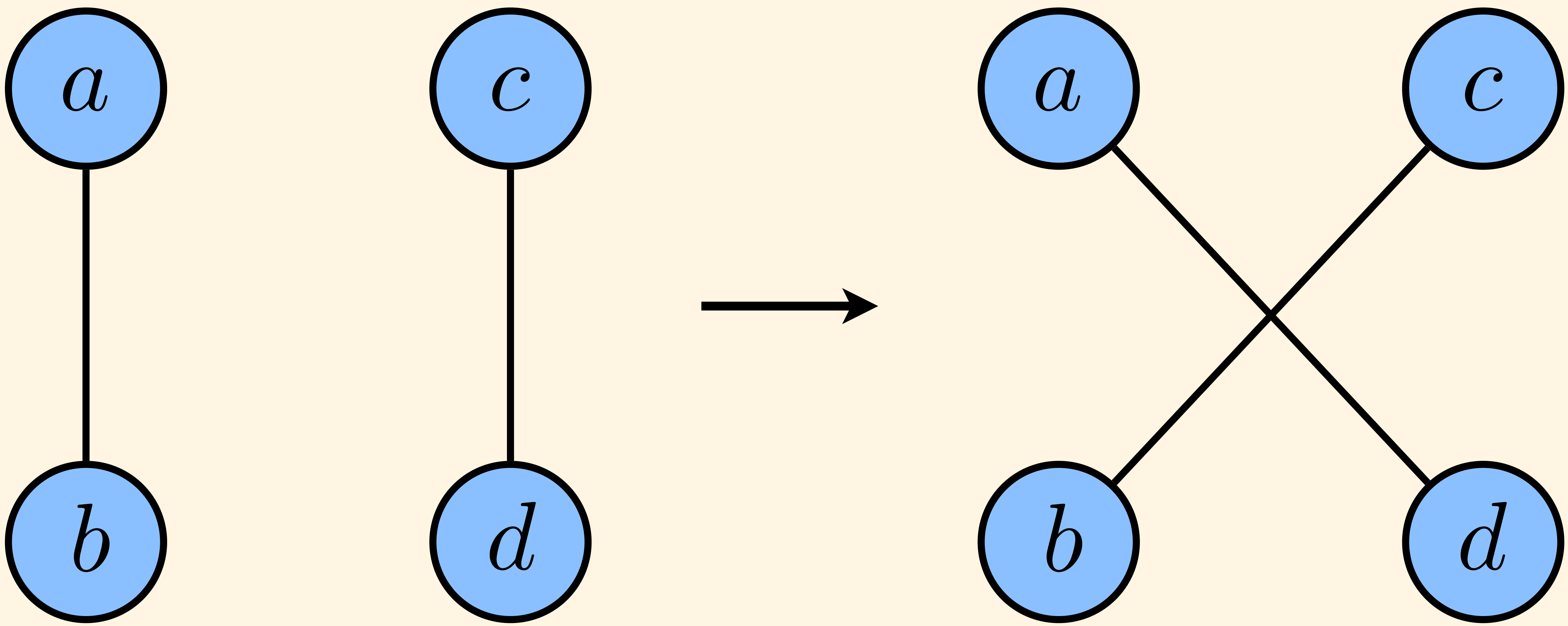
Assumption : consider of graph of diameter 3

Degree d	Order n				
	16	64	256	4096	10000
3	3 / 2.200 0.000%	5 / 3.770 0.211%	8 / 5.636 0.861%	13 / 9.787 2.928%	15 / 11.122 3.225%
4	3 / 1.750 0.962%[2]	4 / 2.869 0.417%	6 / 4.134 1.065%	9 / 6.756 4.423%	10 / 7.601 3.480%
16	N/A	2 / 1.746 0.000%	3 / 2.093 8.026%[2]	4 / 3.254 8.768%	5 / 3.626 1.072%
23	N/A	2 / 1.635 0.000%[1]	2 / 1.910 0.000%	4 / 2.887 0.752%	4 / 3.201 8.697%
60	N/A	2 / 1.048 0.000%[1]	2 / 1.765 0.000%[1]	3 / 2.295 8.976%	3 / 2.650 0.624%
64	N/A	N/A	2 / 1.749 0.000%[1]	3 / 2.242 12.994%[2]	3 / 2.610 1.012%

Outline of Local Search



Neighbor (switch)



Take two edges

Exchange endpoints

Evaluation

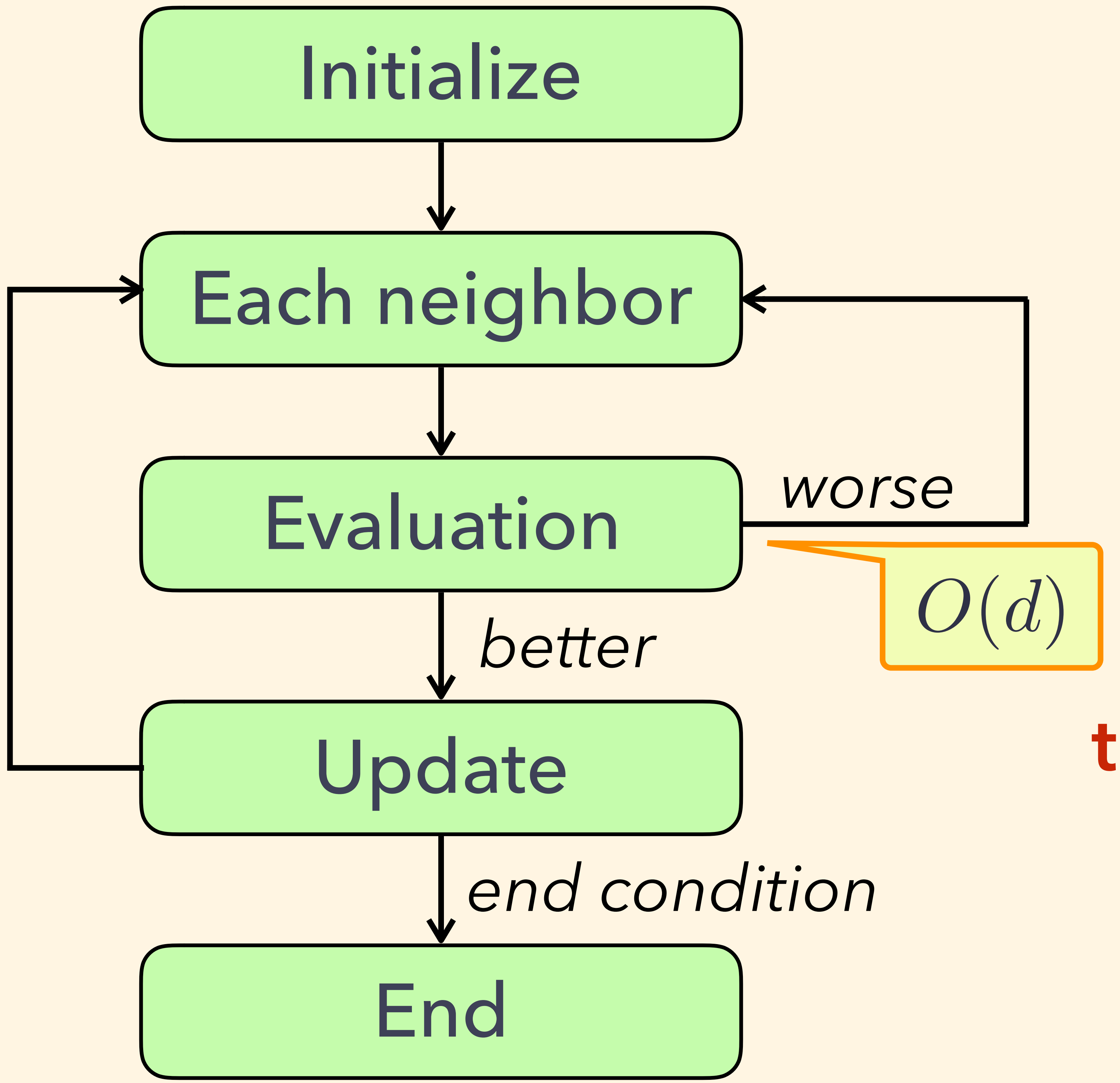
- Evaluation : ASPL value

→ Shortest path of all node pairs

→ $O(VE) = O(n^2 d)$ (BFS from each node)

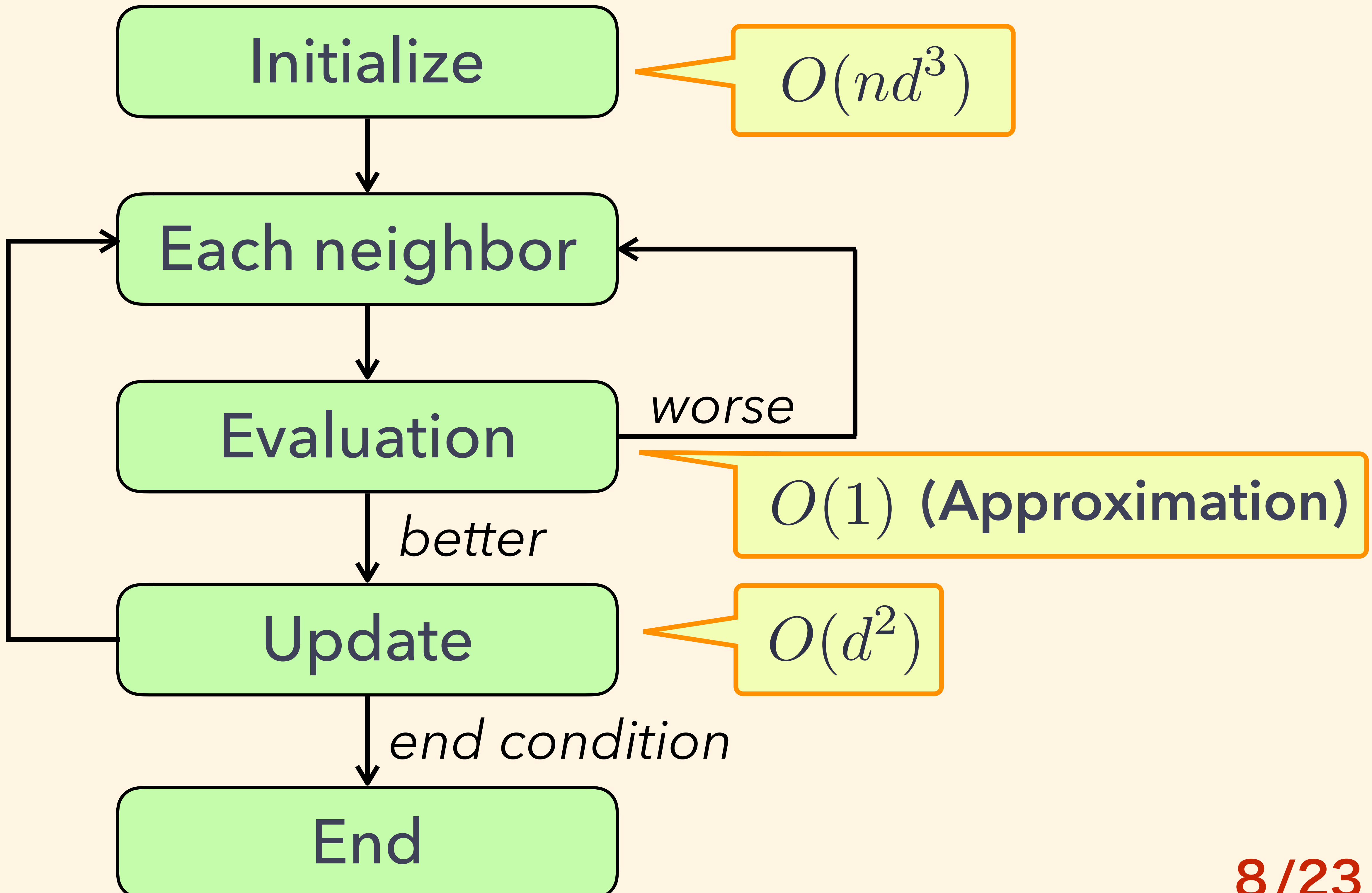
(In fact graph of diameter 3 can be evaluated in $O(d)$ time)

Time complexity (diameter=3)



too slow!!!

Our method (diameter=3)



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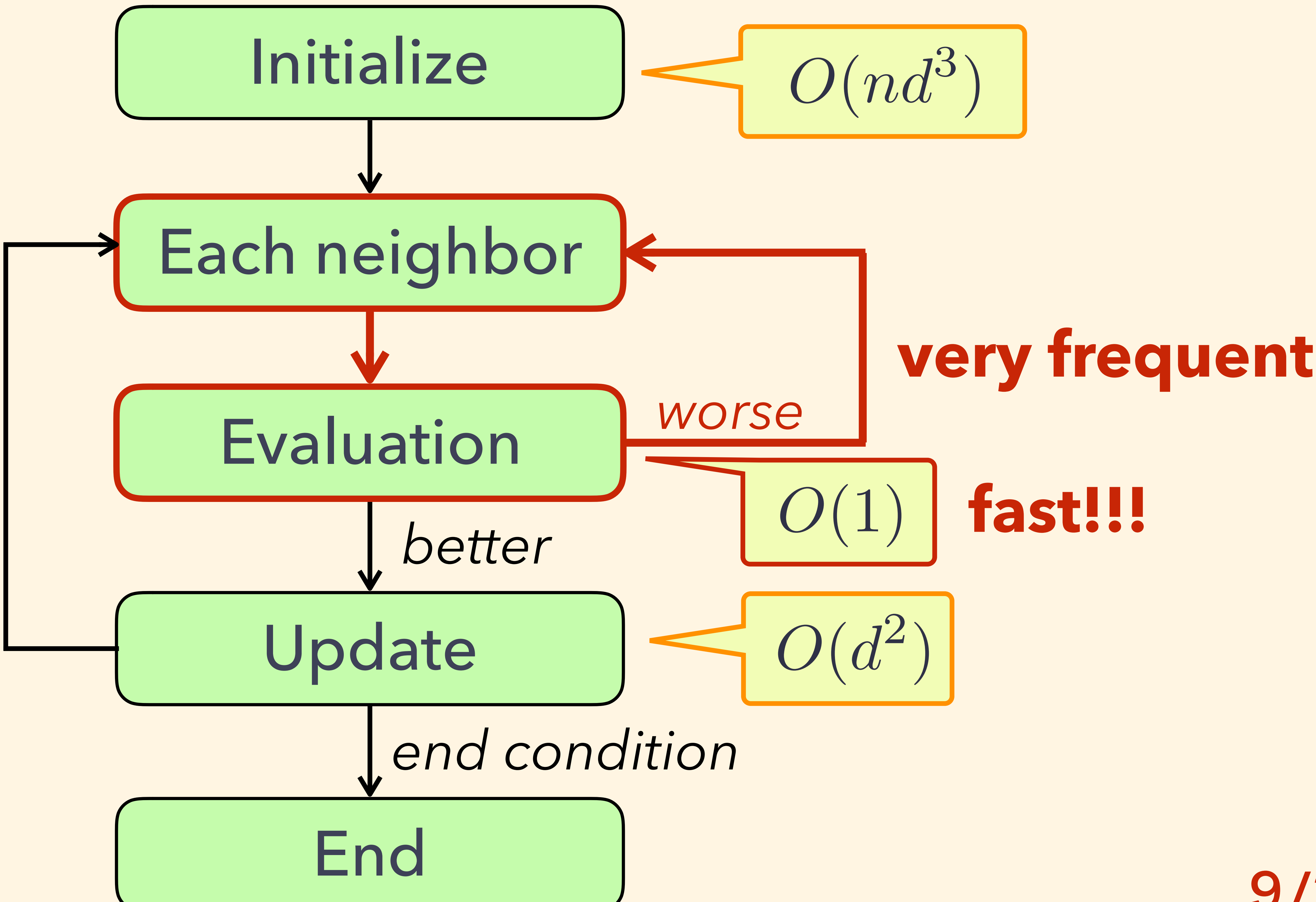
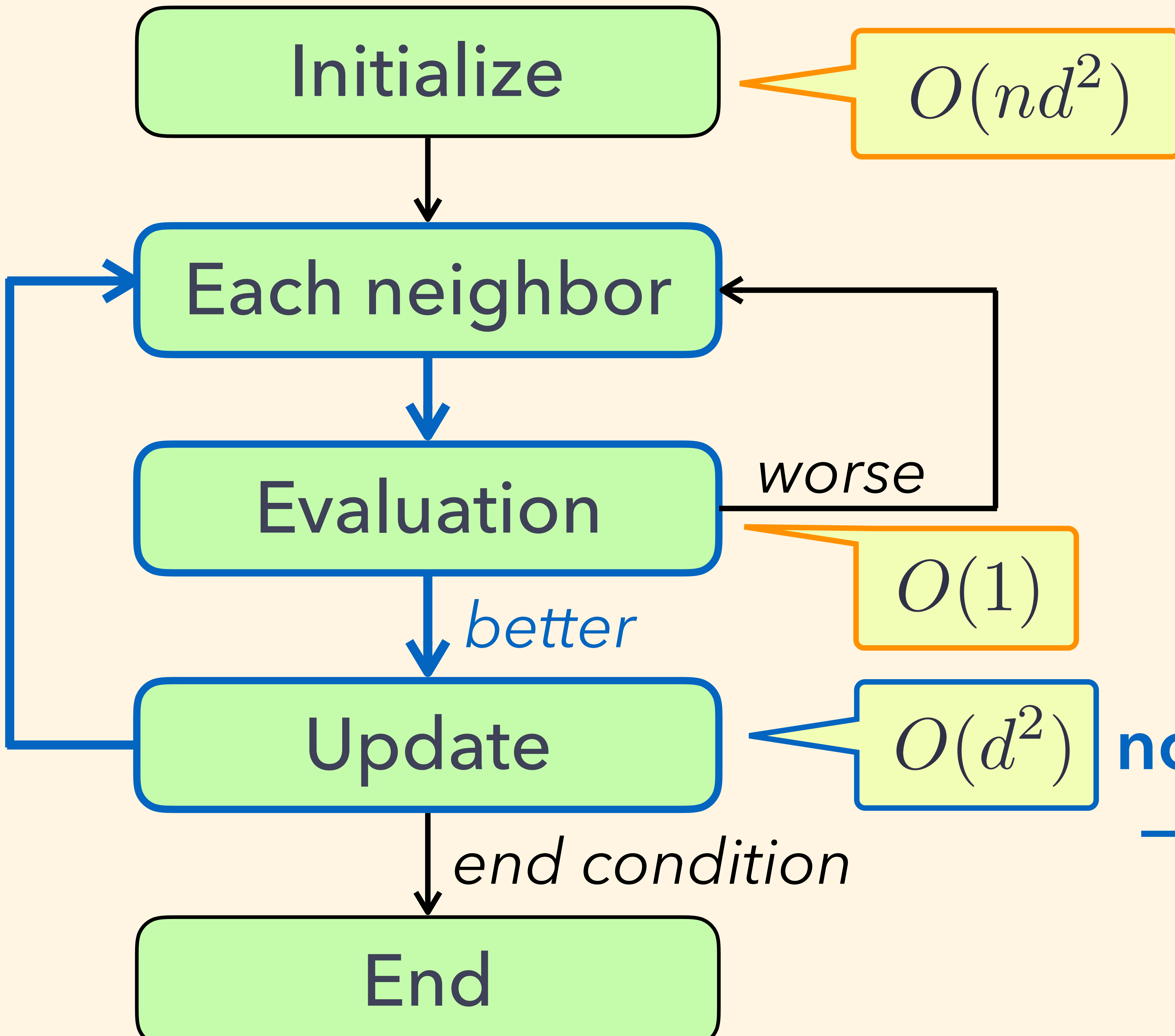


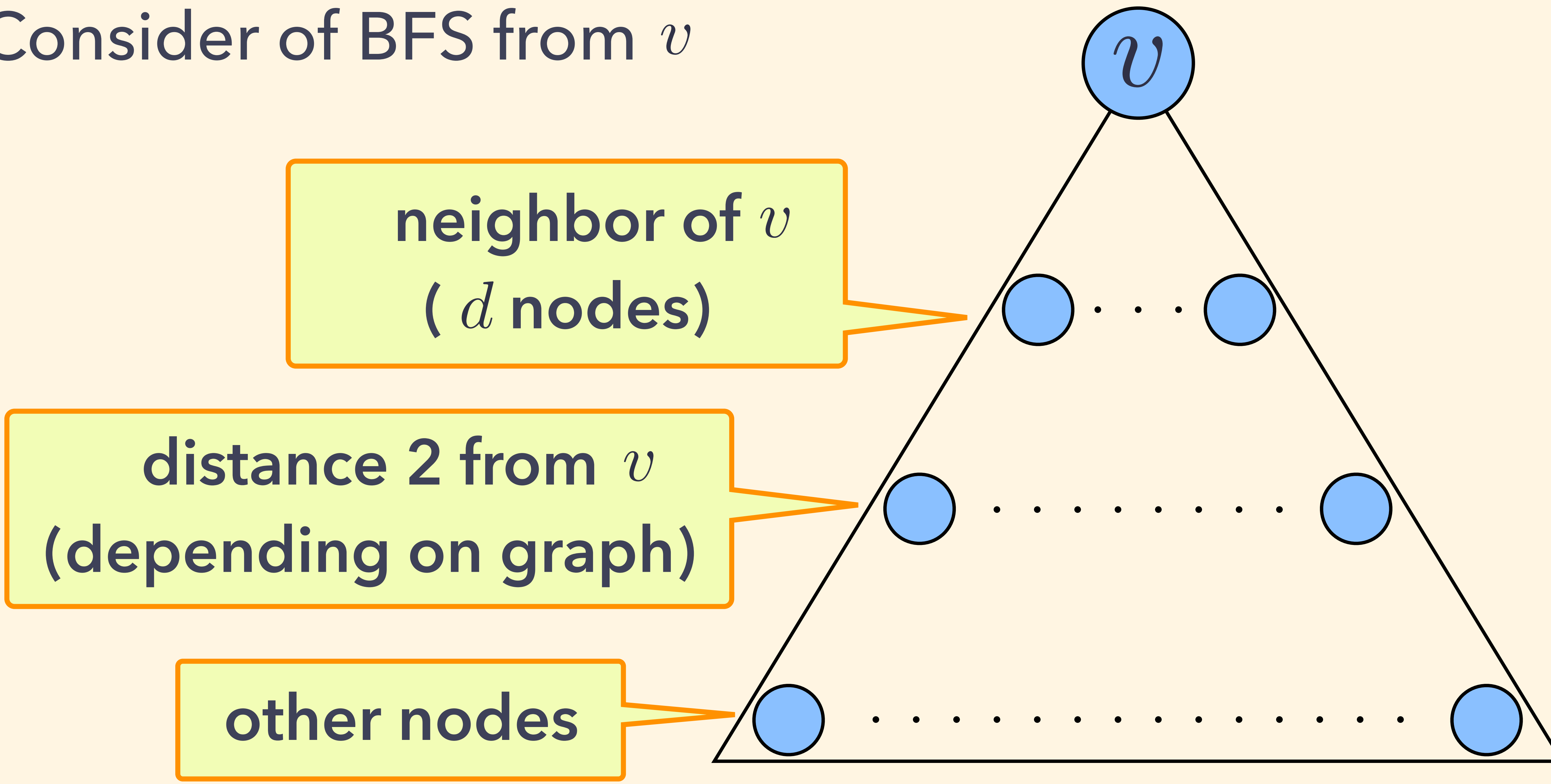
Our method (diameter=3)



not frequent
→not so bad

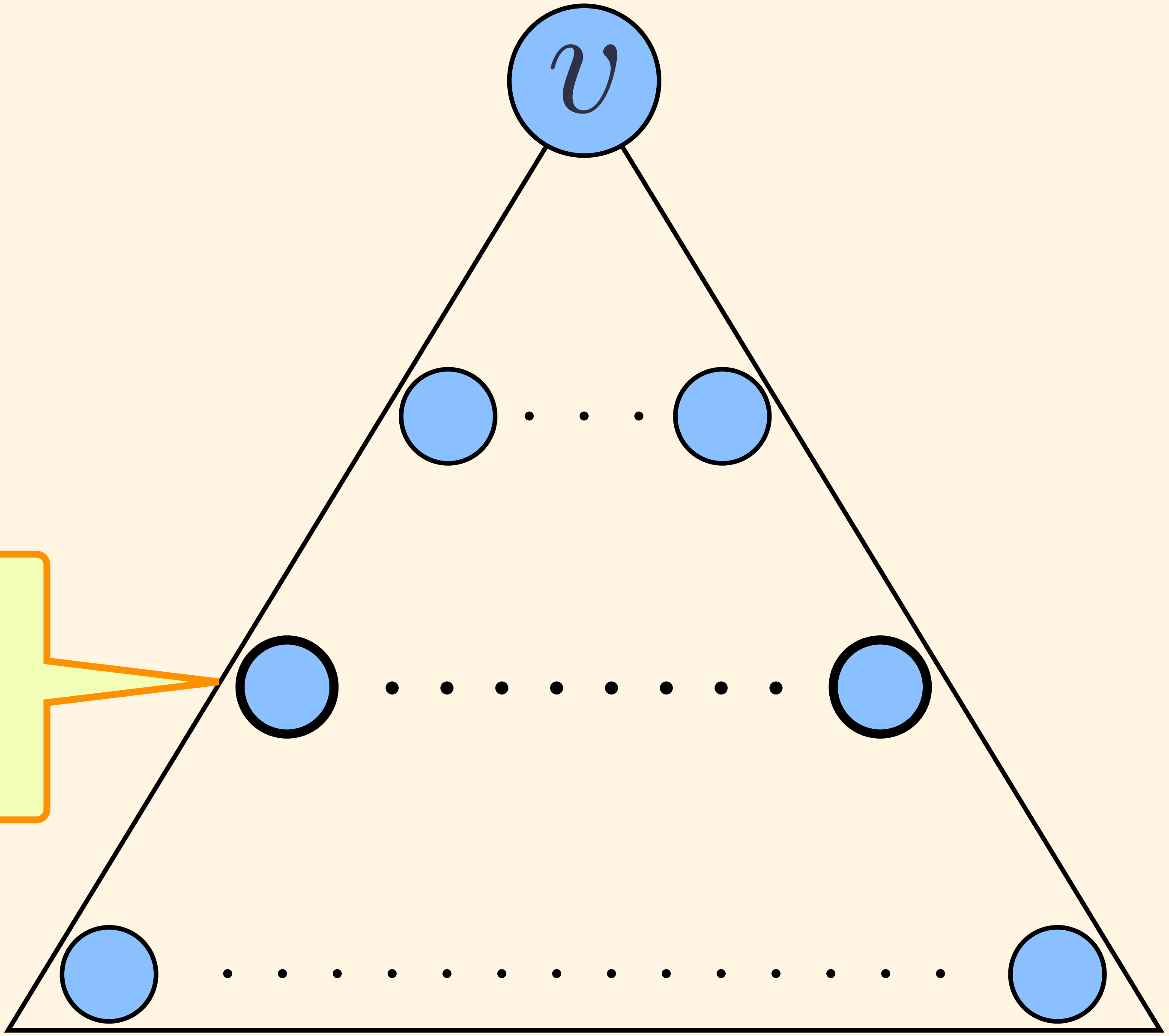
Approach (BFS)

Consider of BFS from v



Approach (BFS)

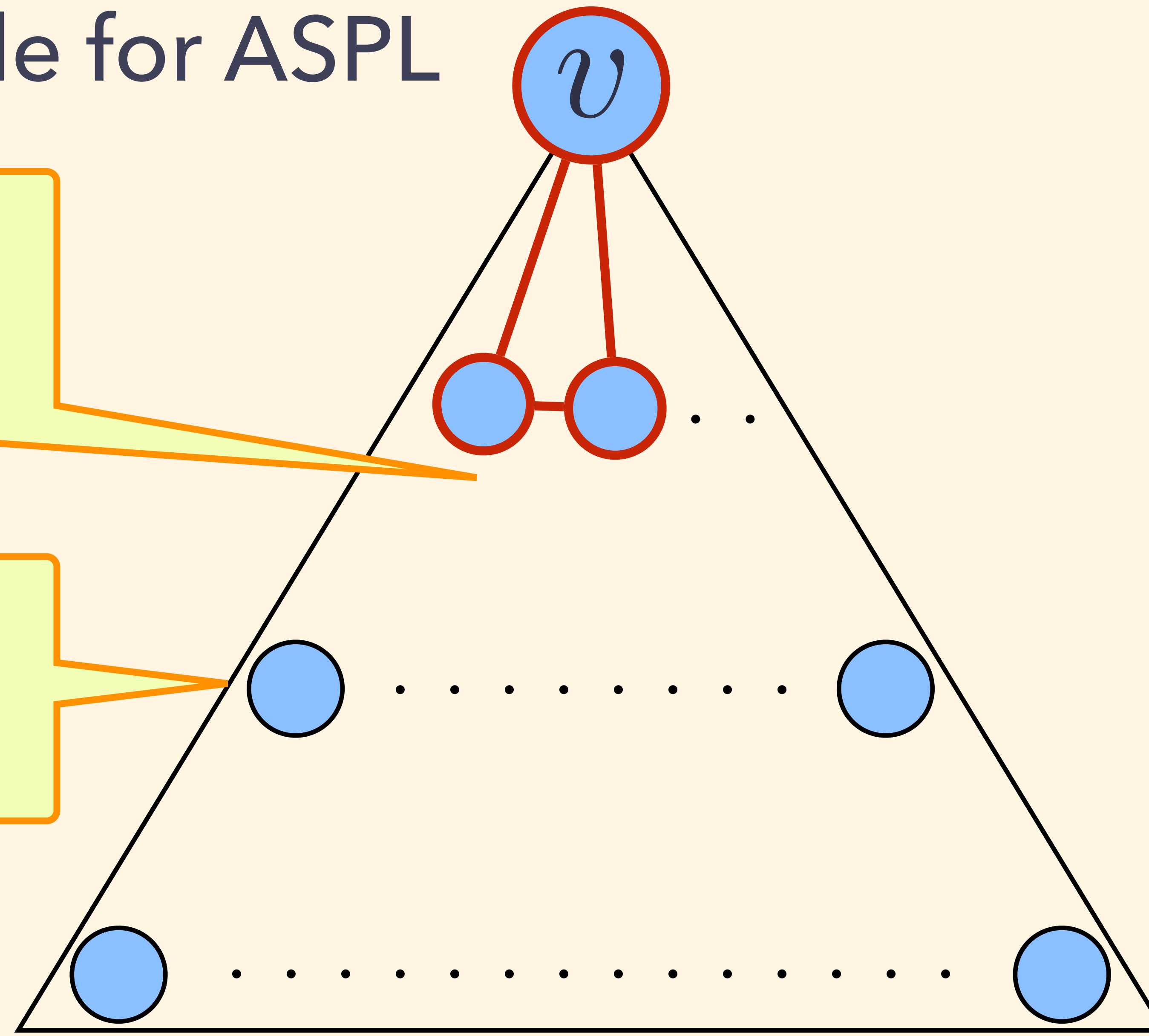
Consider of BFS from v



more these nodes
→ better ASPL

Approach (BFS)

triangles are undesirable for ASPL

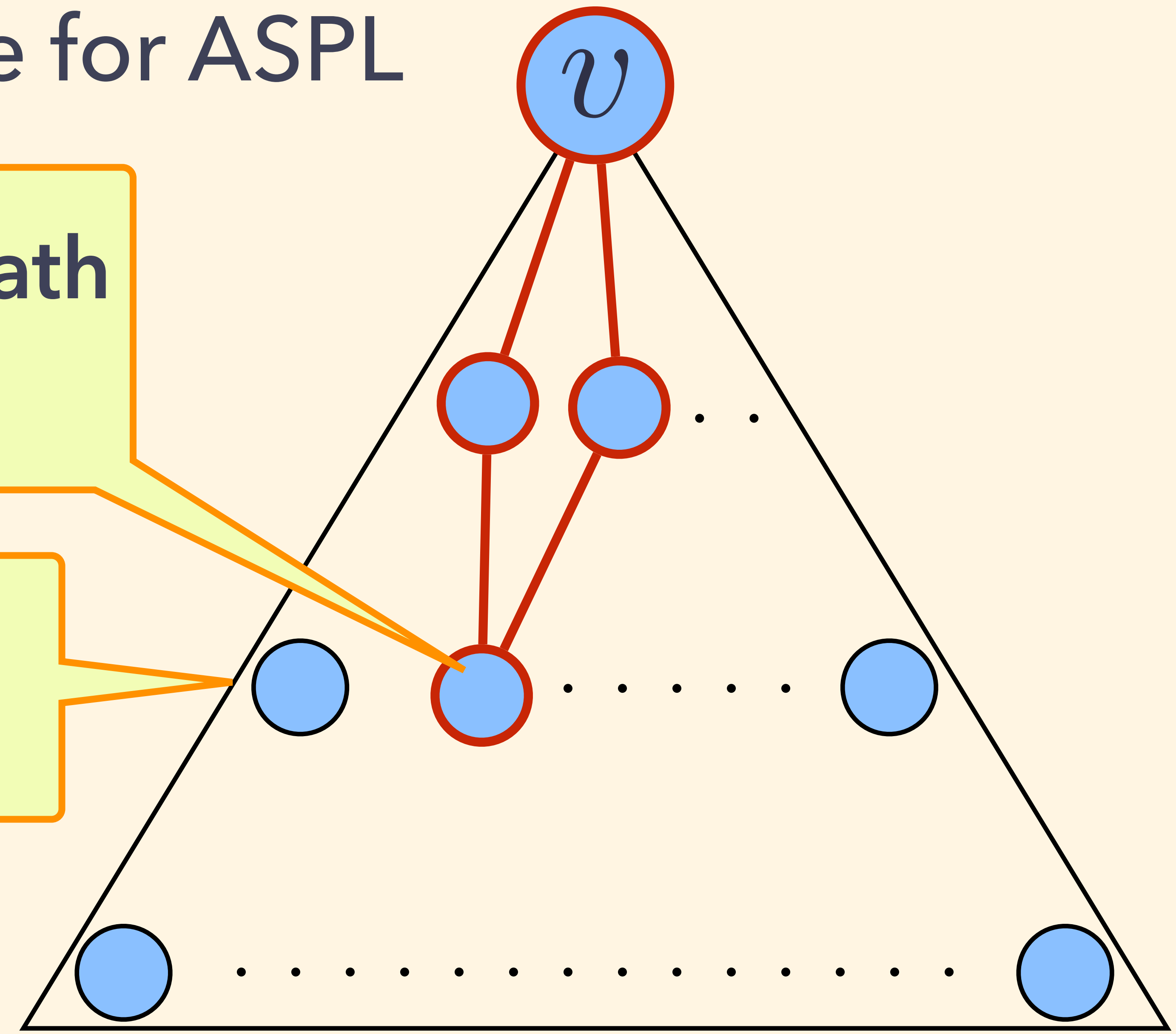


less edges below this triangle

<ins>less</ins> these nodes → <ins>worse</ins> ASPL

Approach (BFS)

squares are undesirable for ASPL



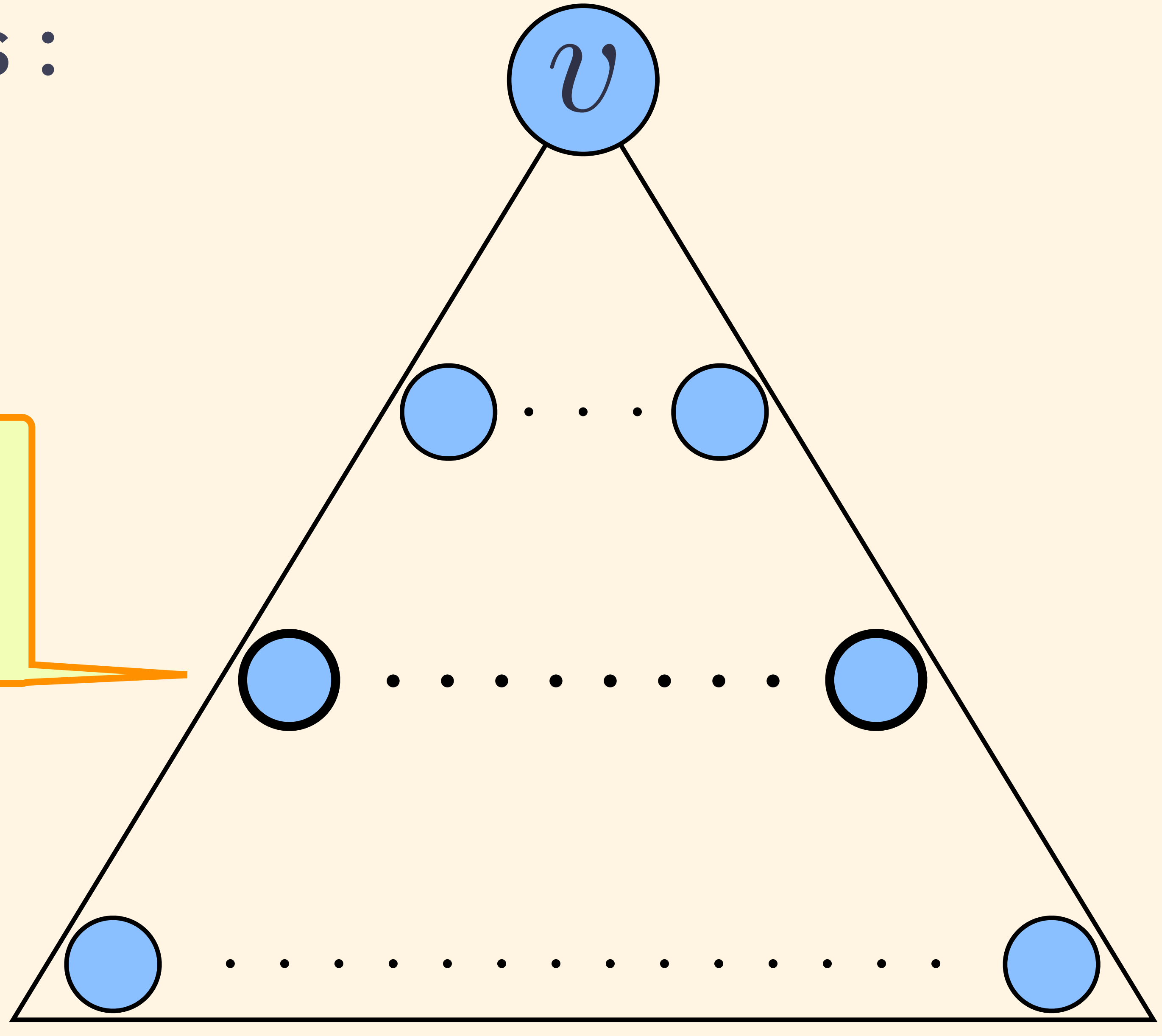
duplication of shortest path
(useless edge)

less these nodes
→ worse ASPL

Approach (BFS)

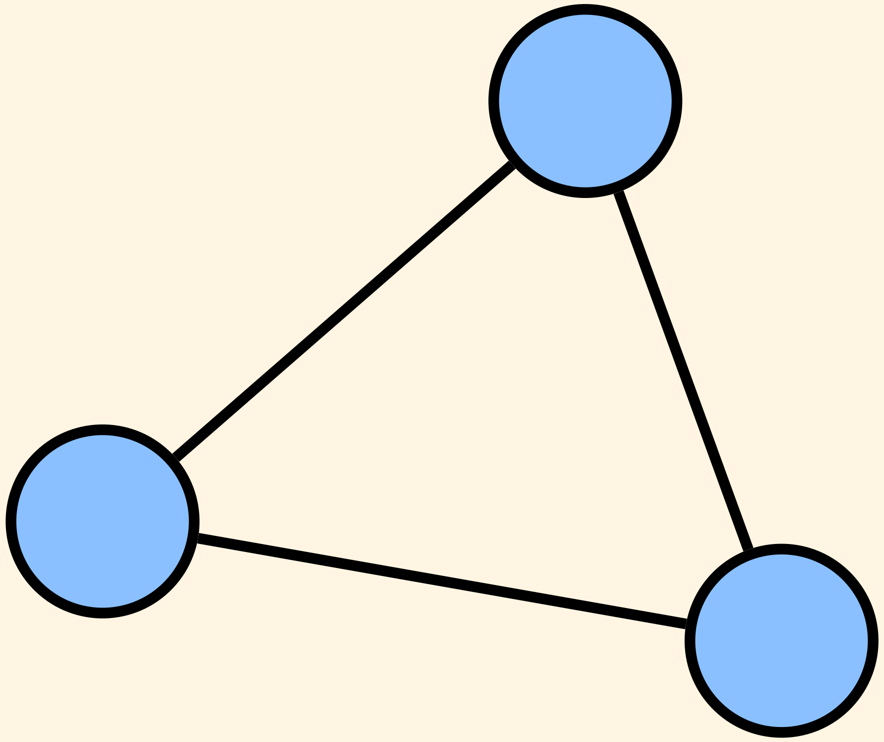
no triangles, no squares :

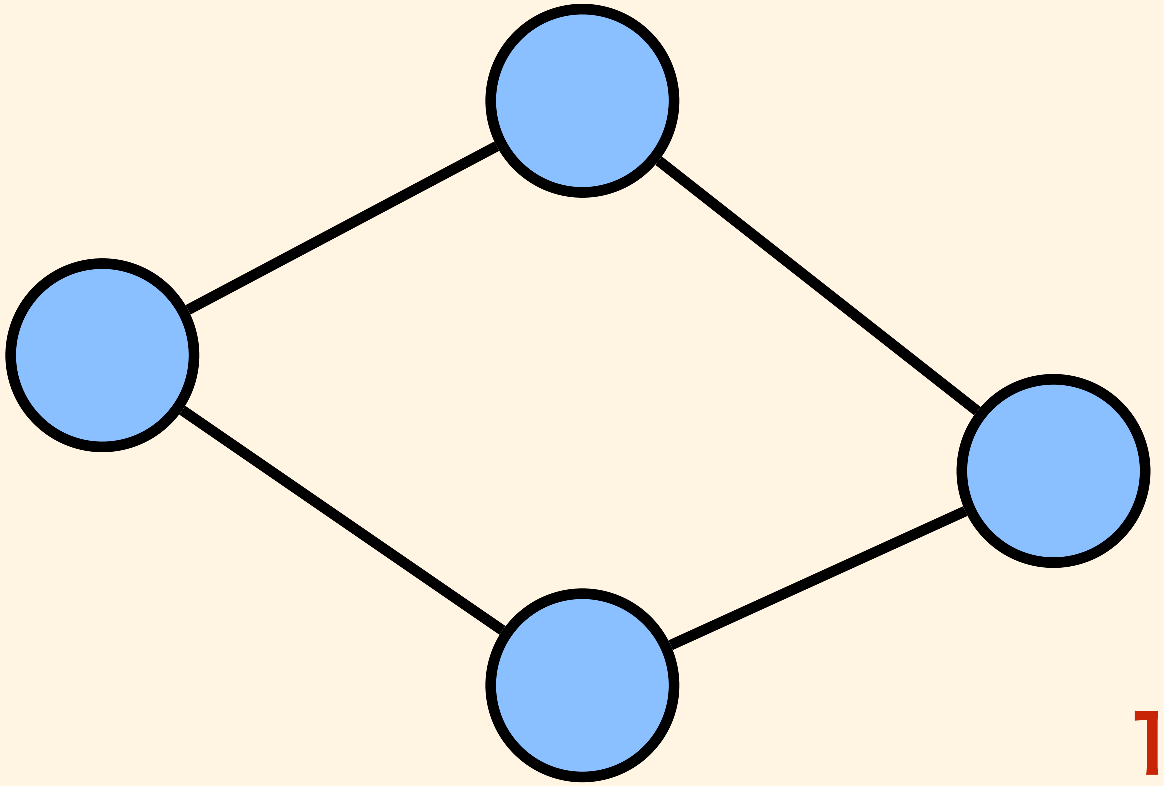
maximum number
→ best ASPL



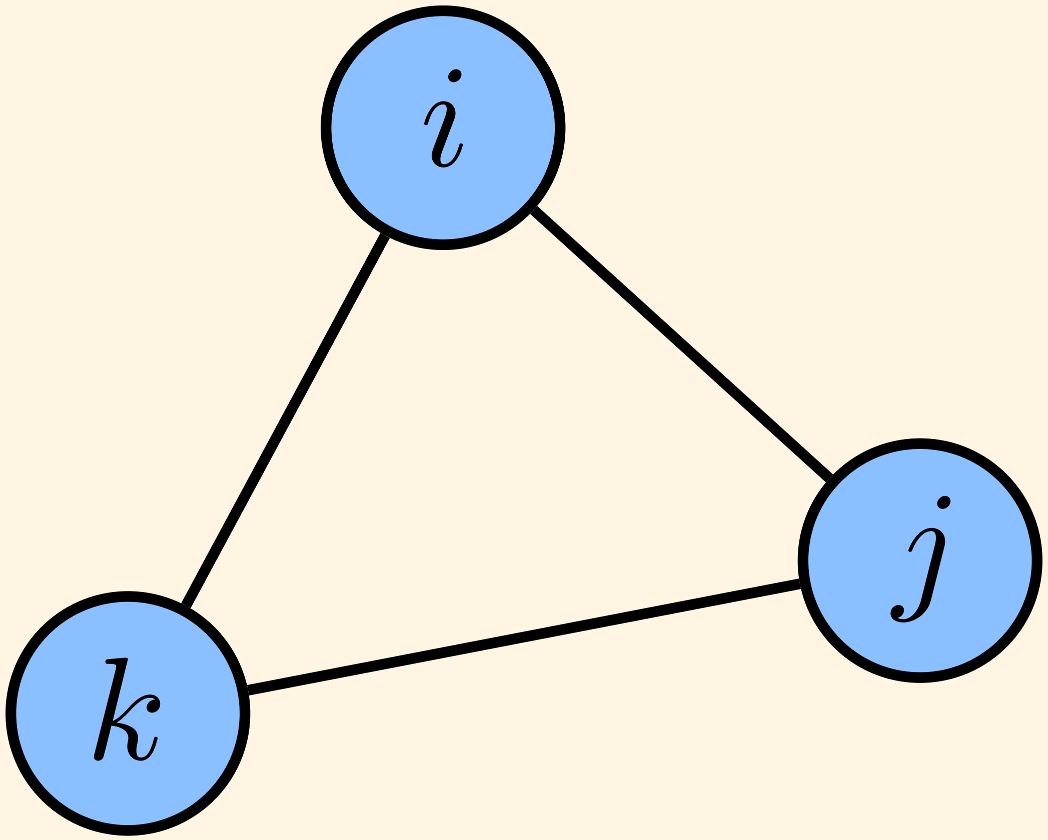
Approach (Triangle, Square)

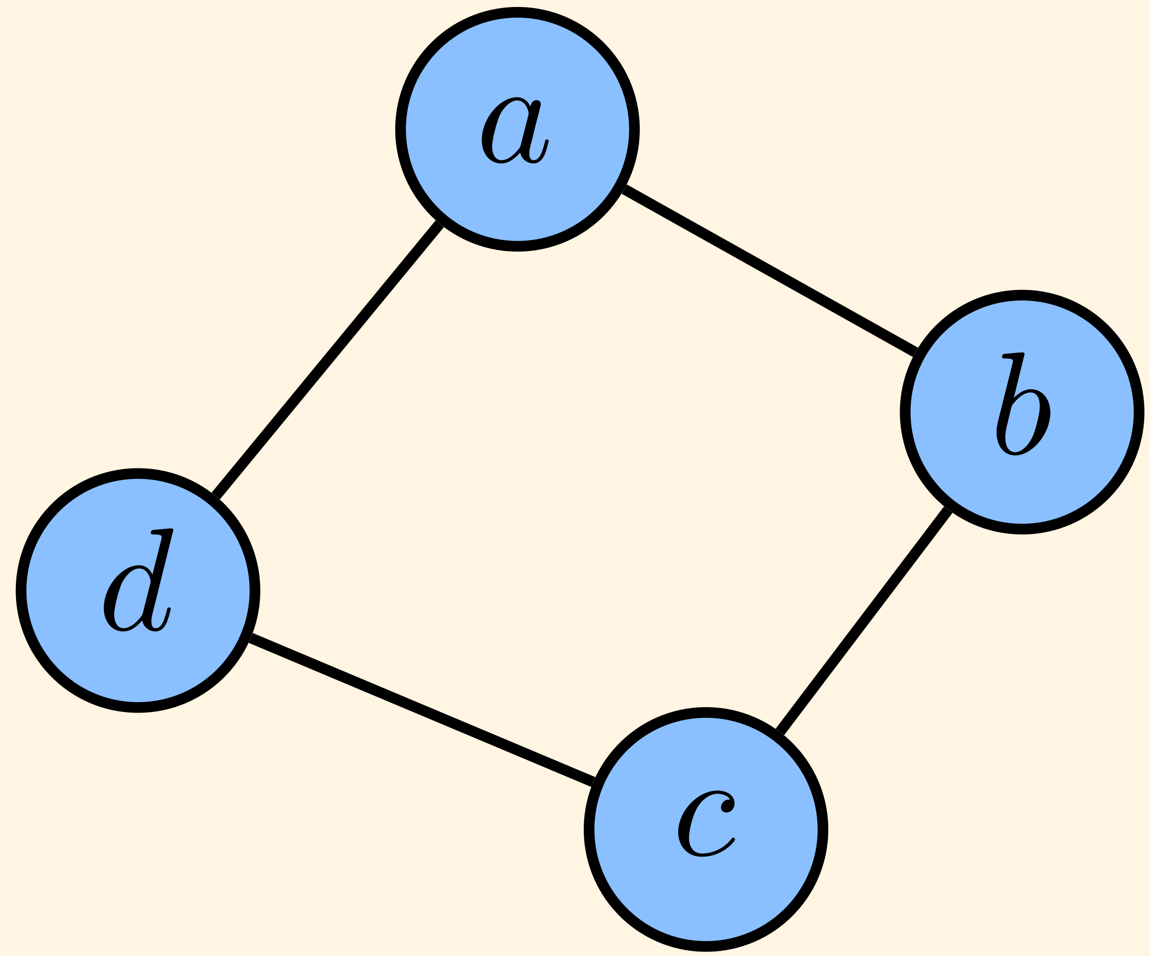
- Reducing triangles and squares is important for low-ASPL graph
- Evaluation by <u>**number of triangles and squares**</u>





Triangle, Square





One Triangle affect 3 pairs :

$$(i, j)(j, k)(k, i)$$

One Square affect 2 pairs :

$$(a, c)(b, d)$$

Main idea

- $\triangle$, $\square$: number of **triangles/squares** in graph
- Evaluation function : $3\triangle + 2\square$
- In fact, under some condition :

$$ASPL \propto \frac{3n(n-1) - nd(d+1)}{2} + 3\triangle + 2\square$$

- Moreover, this is **upper bound** of ASPL :

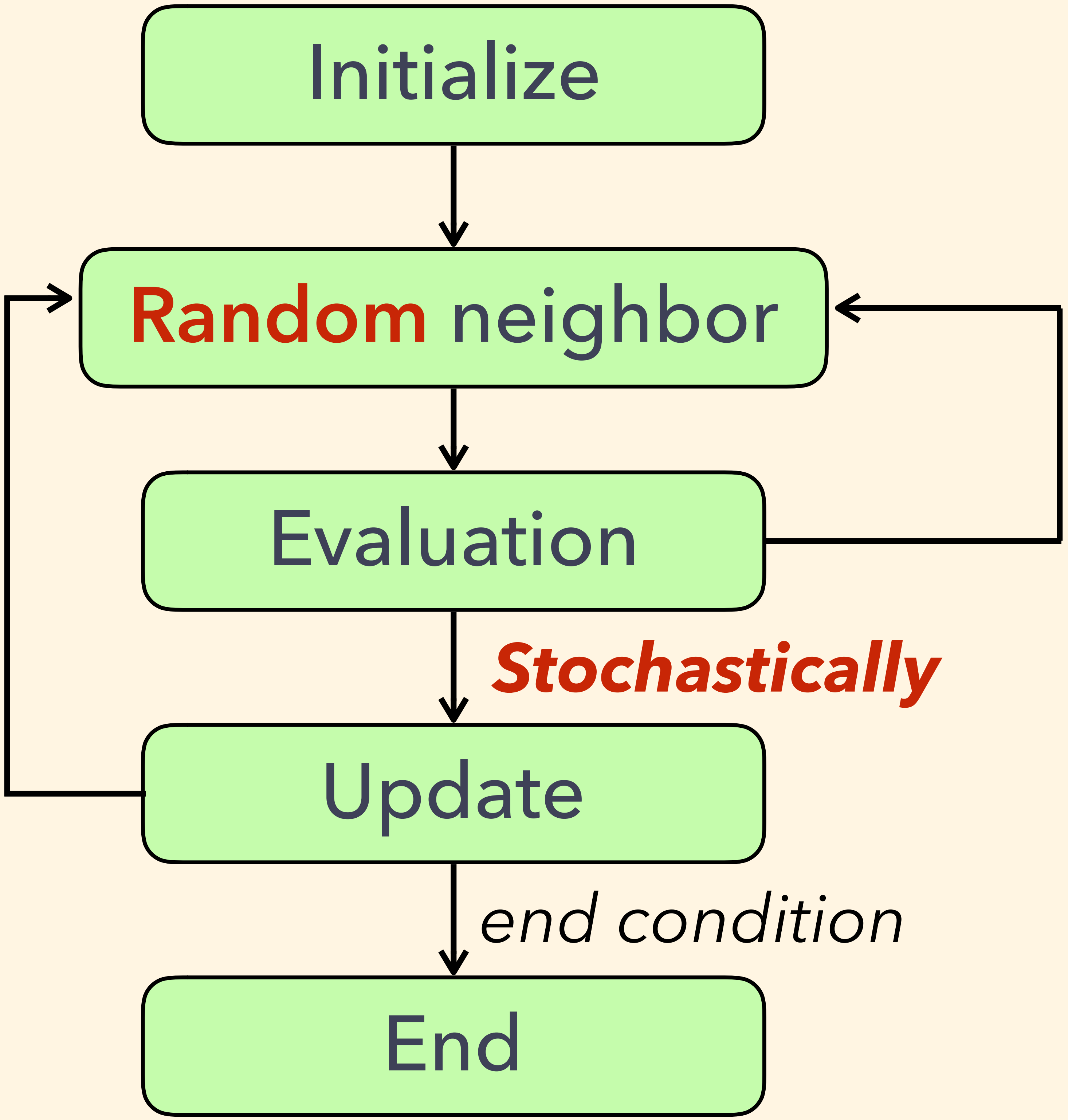
$$ASPL \leq \frac{3n(n-1) - nd(d+1) + 6\triangle + 4\square}{2n(n-1)}$$

Time complexity

$$(A:\ \text{adjacency matrix})$$

- Initialize : prepare tables $A, A^2, A^3 \rightarrow O(nd^3)$
- Evaluation : fluctuation of $\triangle, \square \rightarrow O(1)$
- Update : rewrite tables $\rightarrow O(d^2)$

Application : Simulated Annealing



Simulated Annealing

- Simulated Annealing is better than Greedy
- Evaluation function : $3\triangle + 2\square$
- cool very slowly

Simulated Annealing

Degree d	Order n				
	16	64	256	4096	10000
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We got the best solution for large graph!!!

Conclusion

- ASPL of graph of diameter 3 can be approximated using $\triangle, \square$
- The fluctuation of the triangles/squares in graph can be calculated in $O(1)$ with tables.
- These facts enable us to do Simulated Annealing and to construct low-ASPL graph.

Thank you