Workshop in CANDAR'16, Higashi-hiroshima, Hiroshima, Japan 2016-11-22

# Graph Golf 2016

## The Order/degree Problem Competition

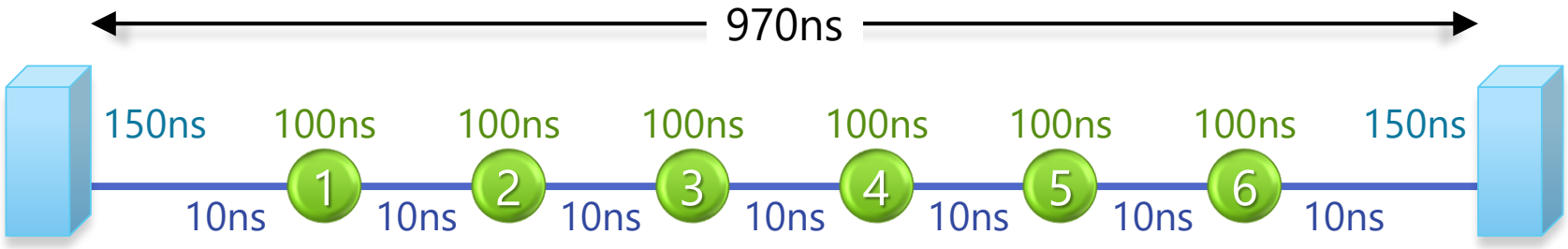
# Timetable

| | |
|---|---|
| 14:00-14:15 | Introduction |
| 14:15-14:25 | Award Ceremony |
| 14:30-15:10 | Keynote by Hiroshi Inoue (IBM Research Tokyo) |
| 15:10-15:50 | Talk by Takayuki Matsuzaki (Kumamoto Univ.) |
| 15:50-16:30 | Talk by Ryosuke Mizuno (Kyoto Univ.) |
| 16:30-17:00 | Discussion |

# Introduction

# Background

- Supercomputer's interconnects are latency-sensitive
  - 1µs for 3M cores ≒ **7 hops for 100k nodes**

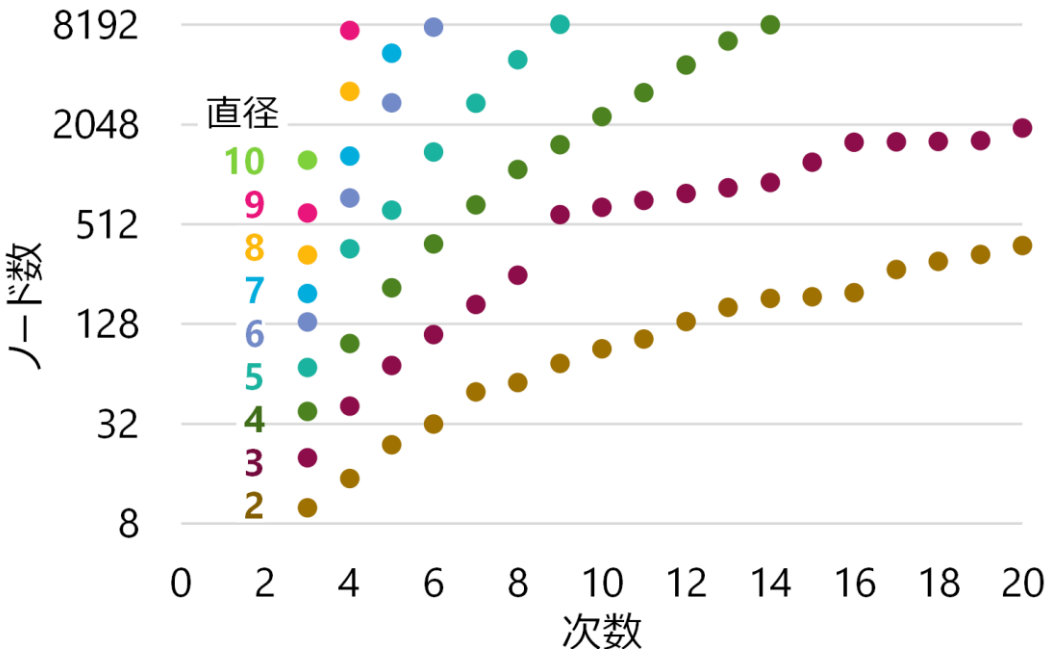



- Random graphs are promising
  - Really? The "best" graph is unique, isn't it?

http://qwe.jp/wvga/edg.jpg-%E3%82%A4%E3%83%BC%E3%83%8E%E3%83%83%E3%82%AF

# Degree/Diameter Problem (DDP)

- Given a diameter $k$ and a degree $d$, find a graph with the largest order $n$
  - Known solutions are listed in the *Combinatorial Wiki*
- Solutions' orders are not convenient for hardware



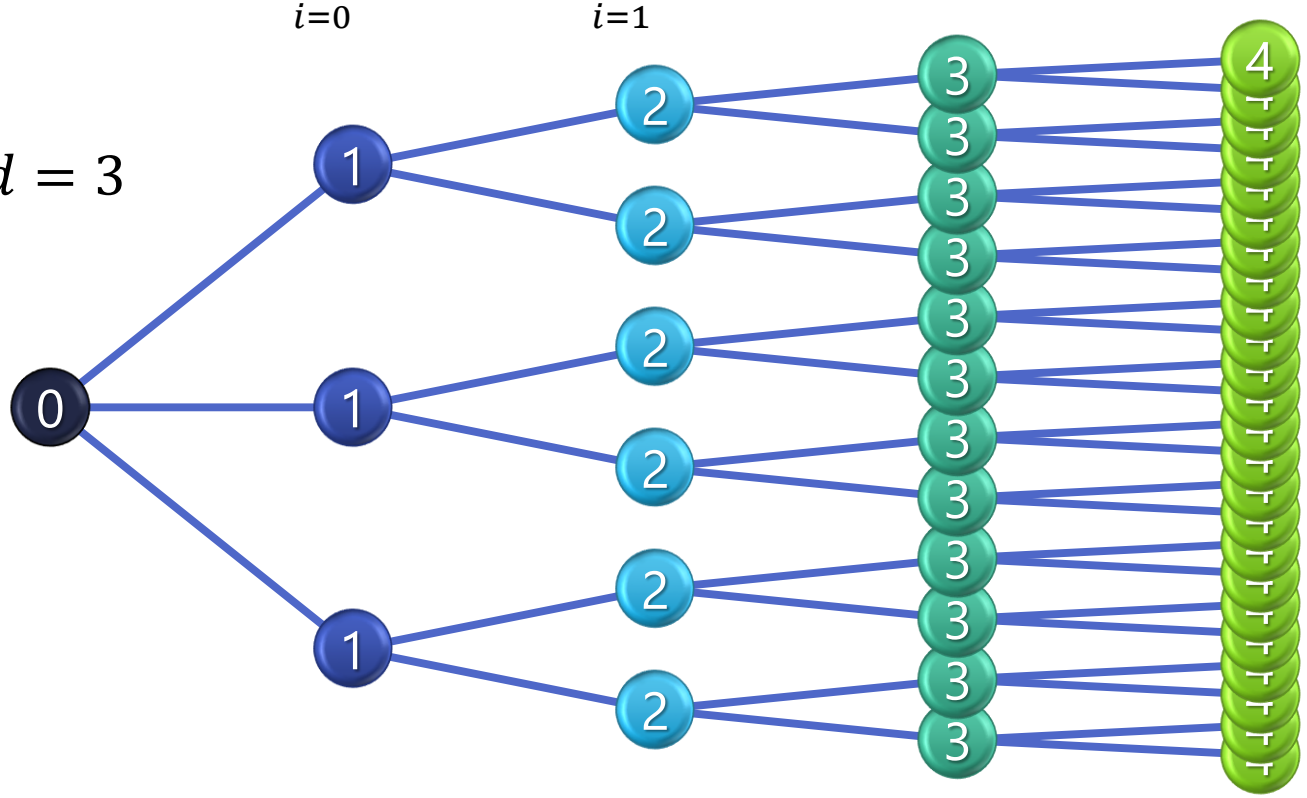

**DDP solutions can not directly be applied to interconnects**

# Moore Bound

- Given diameter $k$ and degree $d$, the upper bound of the order $n$ is

$$N_{d,k} = \sum_{i=0}^{k} n_i = 1 + d \sum_{i=1}^{k} (d-1)^{i-1}$$

- Example for $d = 3$



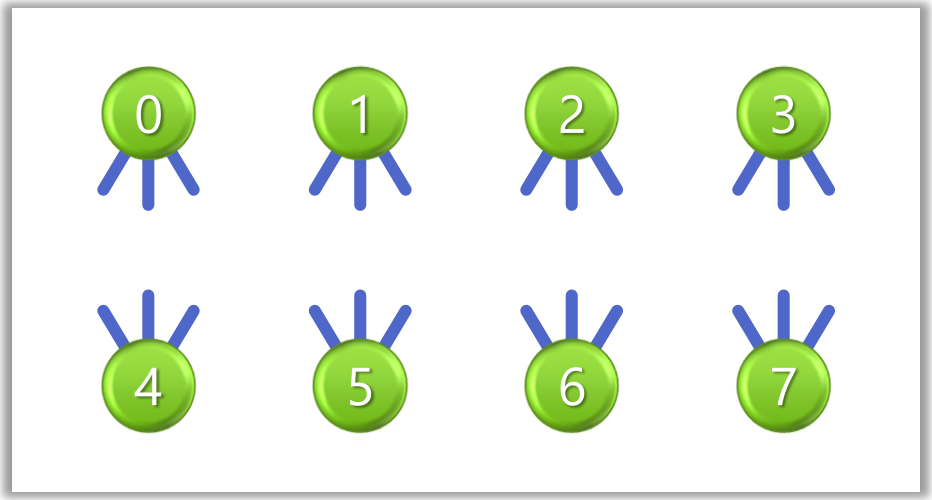

| Diam. $k =$ | | | | |
|---|---|---|---|---|
| Order $n_k =$ | 1 | 3 | 6 | 12 | 24 |
| U.B. $N_{d,k} =$ | 1 | 4 | 10 | 22 | 46 |

# Order/Degree Problem (ODP)

- Given an order $n$ and a degree $d$, find the graph with the smallest diameter $k$.
  - Among those with the same diameter, find the one with the smallest ASPL $l$.

A straightforward solution to design interconnects

- Practice: $n = 8, d = 3$



# Moore Bound

- The lower bound $K_{n,d}$ of diameter $k$ for order $n$ and degree $d$

$$K_{n,d} = \begin{cases} \left\lceil \frac{n-1}{2} \right\rceil & \text{if } d = 2 \\ \left\lceil \log_{d-1}\left( \frac{(n-1)(d-2)}{d} + 1 \right) \right\rceil & \text{if } d > 2 \end{cases}$$

- The lower bound $L_{n,d}$ of ASPL $l$ for order $n$ and degree $d$

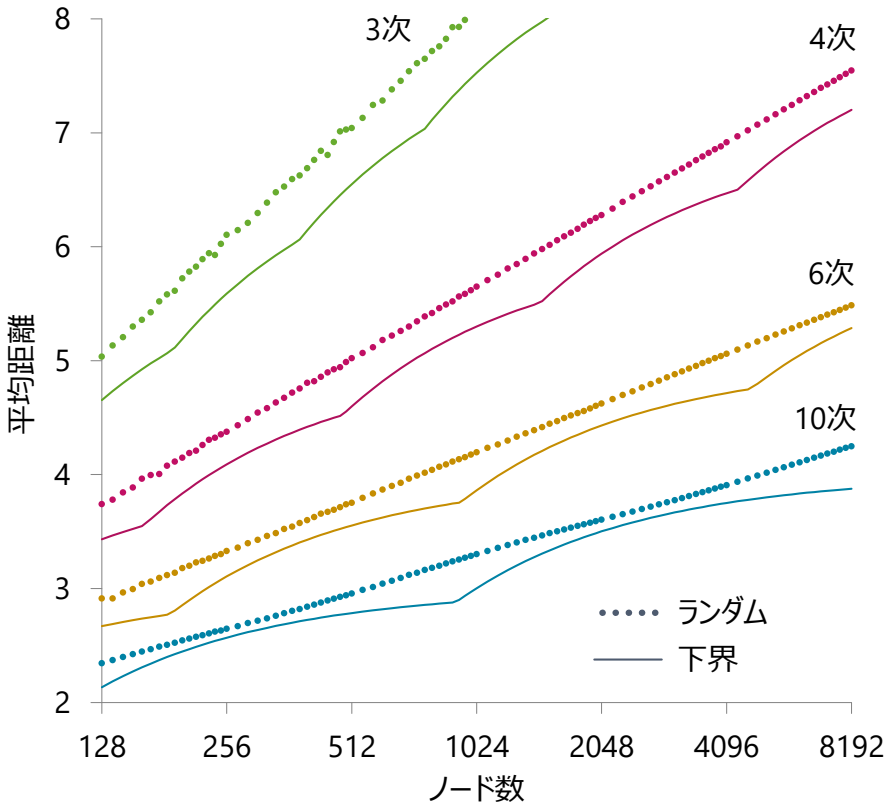
$$L_{n,d} = \begin{cases} 1 & \text{if } K_{n,d} = 1 \\ \frac{\sum_{i=1}^{K_{n,d}-1} id(d-1)^{i-1} + K_{n,d}\left(n - 1 - \sum_{i=1}^{K_{n,d}-1} d(d-1)^{i-1}\right)}{n-1} & \text{if } K_{n,d} \geq 2 \end{cases}$$

- ASPL gap $p$

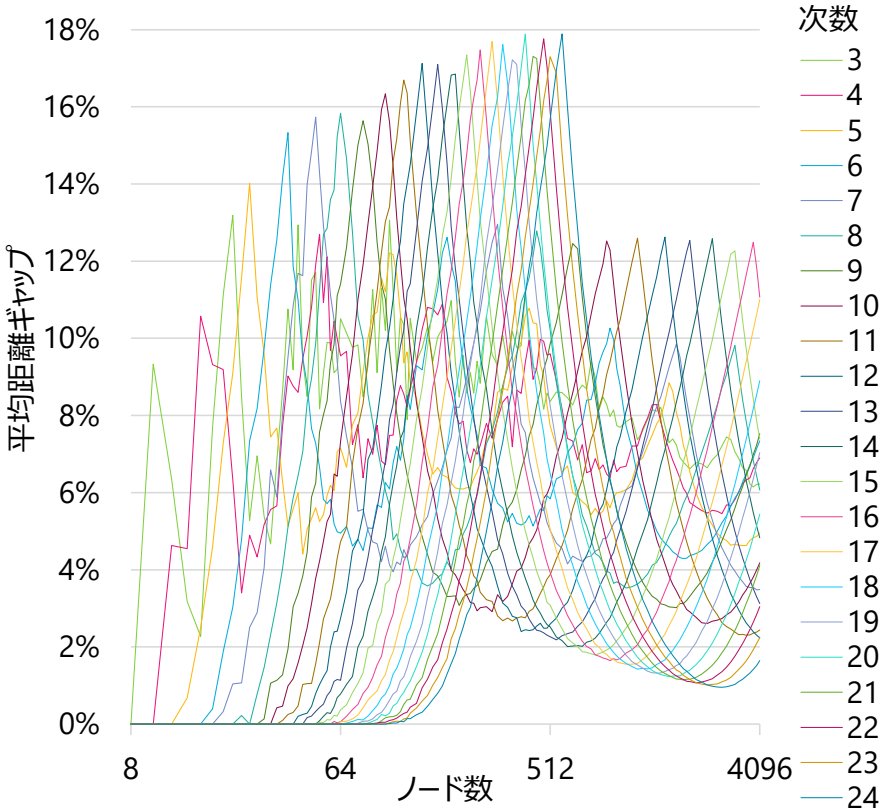
$$p = \frac{l - L_{n,d}}{L_{n,d}}$$

# Don’t Random Graphs Suffice?

ASPL vs. lower bound [hops]




ASPL gap [%]

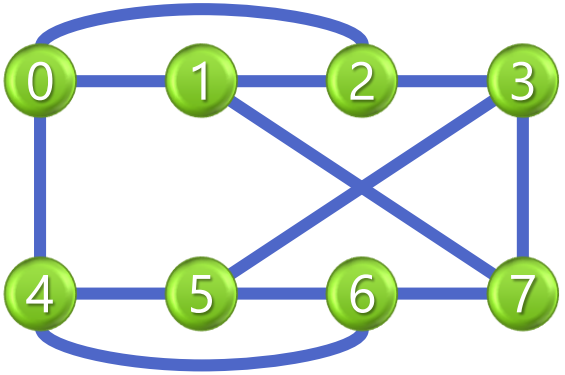



**There can be such graphs better than the random graphs**

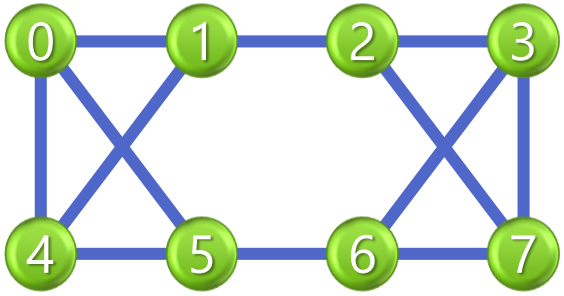
# Practice - Answers

Lower bound of ASPL $L_{n,d} = 1.571$

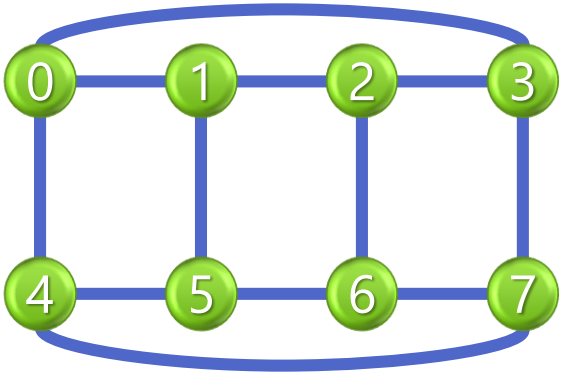

$k = 2$
$l = 1.571$
$p = 0\%$




$k = 3$
$l = 1.643$
$p = 4.55\%$




$k = 3$
$l = 1.786$
$p = 13.6\%$

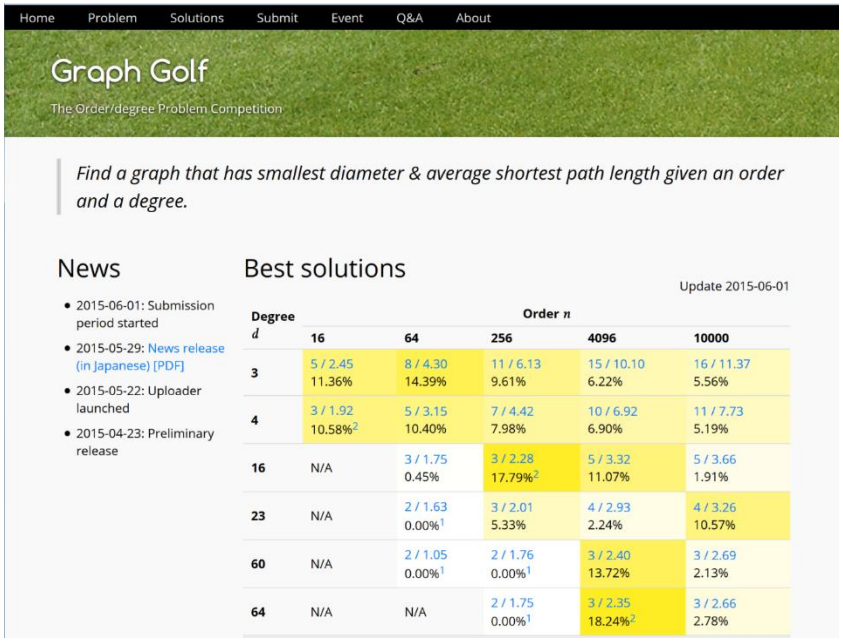



$k = 3$
$l = 1.714$
$p = 9.09\%$

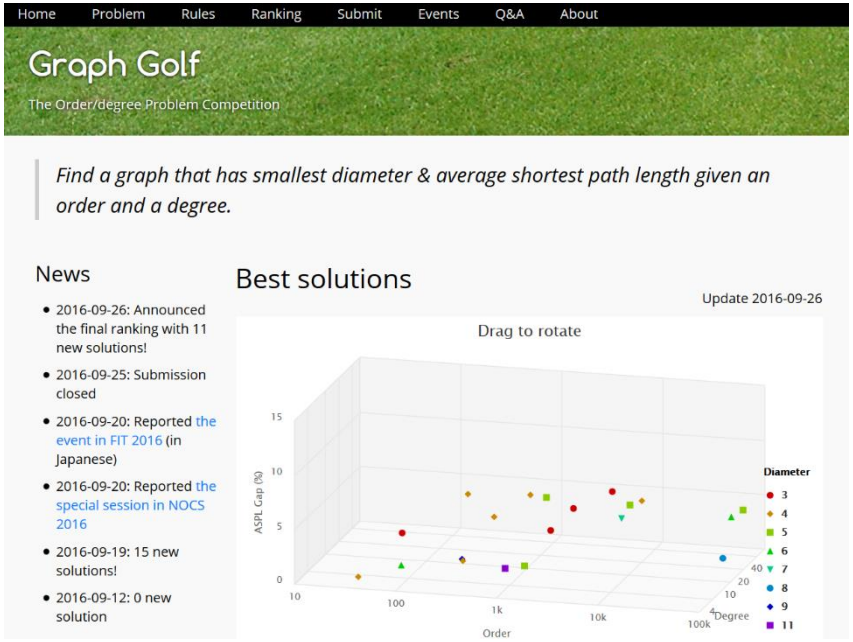
# Graph Golf

- An online competition for the order/degree problem held every year from 2015
- Aims at building a catalog of “best” graphs for each order/degree pair

2015: 25 conditions




2016: 20 conditions



# Award Rules


## Widest Improvement Award

- Who find the largest number of "best" solutions
- The best solution means a graph with the smallest diameter, and with the smallest ASPL among those with the same diameter, for each order/degree pair.


## Deepest Improvement Award

- Who achieve the largest improvement in ASPL gap over all the order/degree pairs
- The improvement is calculated as $(l_a - l_b) \div L$, where
  $l_a$ = the achieved ASPL
  $l_b$ = ASPL of a random graph
  $L$ = lower bound of ASPL

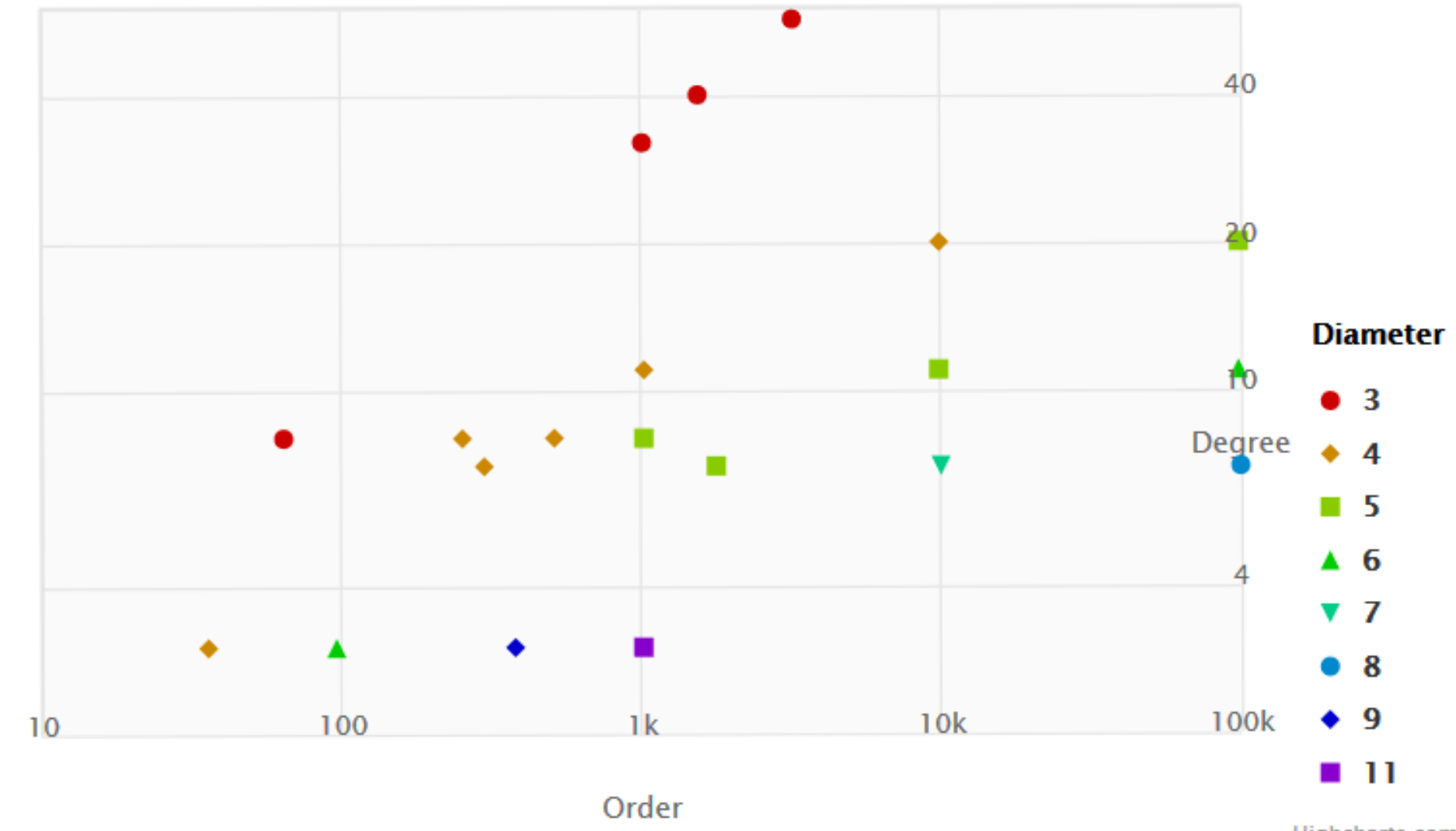
http://freedesignfile.com/9420-trophy-cup-and-medals-vector-set-05

# Publicities

- The 10th IEEE/ACM International Symposium on Networks-on-Chip (NOCS 2016), Aug. 31
- 第10回情報科学技術フォーラム (FIT 2016), Sep. 7
- NII Press Releases
  - Issued today for this event
    http://www.nii.ac.jp/news/2016/1122/

# 2016 Final Results

# Featured Order/degree pairs



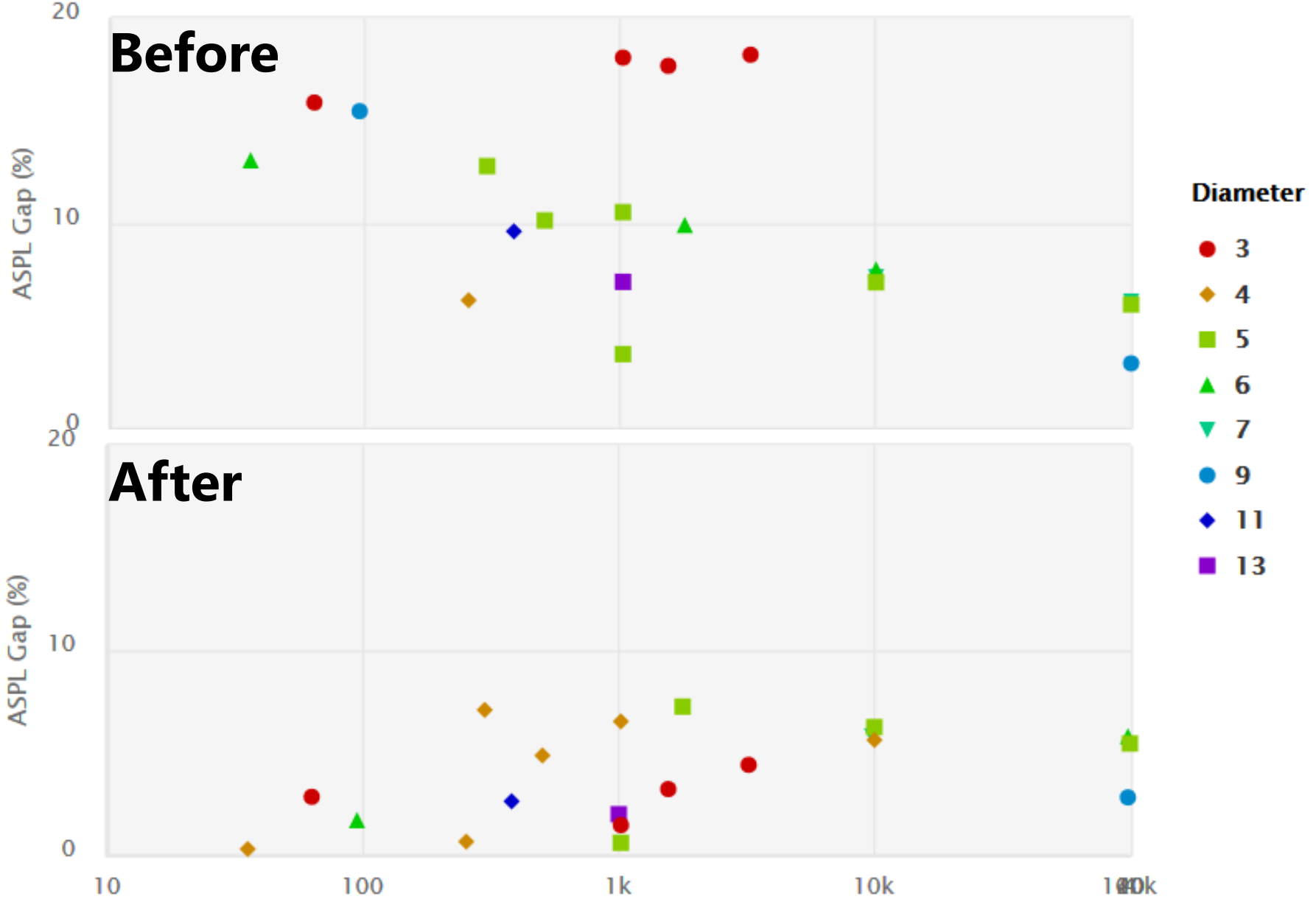
# Widest Improvements

| | Author(s) | Number of best solutions |
|---|---|---|
| 1 | Takayuki Matsuzaki & Teruaki Kitasuka & Masahiro Iida | 10 |
| 2 | H. Inoue | 6 |
| 3 | Ryuhei Mori | 5 |
| 4 | Akira Suzuki & Students in my laboratory | 1 |
| 4 | Ryo Ashida | 1 |
| 4 | Yawara Ishida & Ryosuke Mizuno | 1 |

# Deepest Improvements

| | Author(s) | Improvement |
|---|---|---|
| 1 | Yawara Ishida & Ryosuke Mizuno | 0.16597392471449857 |
| 2 | Takayuki Matsuzaki & Teruaki Kitasuka & Masahiro Iida | 0.15721492521102287 |
| 3 | Ryuhei Mori | 0.1374791144527988 |
| 4 | H. Inoue | 0.13664369256474523 |
| 5 | Akira Suzuki & Students in my laboratory | 0.1309523809523809 |
| 6 | Nobushimi | 0.1304824561403509 |
| 7 | Ryo Ashida | 0.12980269989615797 |
| 8 | Y. SATOTANI | 0.1277258566978193 |
| 9 | thai9cdb | 0.1272066458982347 |
| 10 | Darsein | 0.08024364406779662 |

# Improvement of ASPL Gap



# Award Ceremony

# *Widest Improvement Award*


Takayuki Matsuzaki

and

Teruaki Kitasuka

and

Masahiro Iida

Kumamoto University

# *Deepest Improvement Award*


Yawara Ishida

and

Ryosuke Mizuno

Kyoto University

# *Appreciation for Outstanding Commitment*


Hiroshi Inoue

IBM Research, Tokyo

# Toward Graph Golf 2017

- Does the current rule OK?
- What configurations will be interesting?
- How to attract overseas participants?
- Where to hold the competition?
  - Should we take part in a competitive programming site like Kaggle, TopCoder or AtCoder?
- Any ideas to improve Graph Golf?

- Graph Bank: a searchable corpus of useful graphs
  - The solutions in Graph Golf will be registered